Executive Summary

In this project the task was to design a prototype for an efficient soup cooling system for restaurants using gas injection. The project constraints were to minimize cost, size and power use of a thermal cooling system containing 10 - 20 L of soup. Other considerations included FDA specifications in which the soup had to be cooled to 70℉ in 2 hours and 40℉ in a total of 4 hours. Nitrogen gas was used for the gas injection, due to its inert qualities, low boiling point, inexpensiveness, and safety in human consumption. Three gas injection solutions were considered. The first used an intermediate transfer fluid to facilitate sufficient heat transfer from a reservoir of cold gaseous nitrogen to cool the soup. The second, passed cold gaseous nitrogen around an outer container which served as a medium for heat transfer to cool the soup. The third solution injected the gas directly, allowing the gas to rise through and cool the soup. Direct gas injection was the chosen method because it is an efficient process that allows heat transfer to occur directly between the gas and soup. The efficiency of the process results in a cooling time of approximately 30 minutes or just an eighth of the required cooling time set forth by the FDA. The power usage for cooling one load of soup is around 0.035 kWh and the approximated cost for the system is $1,750. This efficiency is accompanied by design choices that capture the human element: handles and bearings to allow for easy pouring of the soup after cooling and a control panel allows the user to insert the desired temperature through the use of a microcontroller which has a friendly user interface. Rigorous thermal analysis coupled with robust design choices resulted in a soup cooling system that far surpassed any of the requirements set forth.

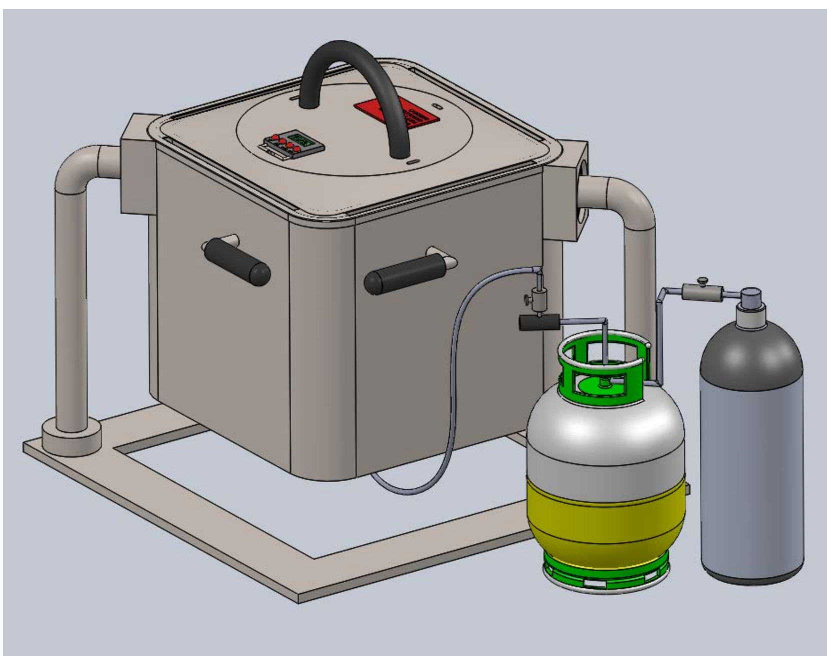

Figure 1: Isometric View of Assembly

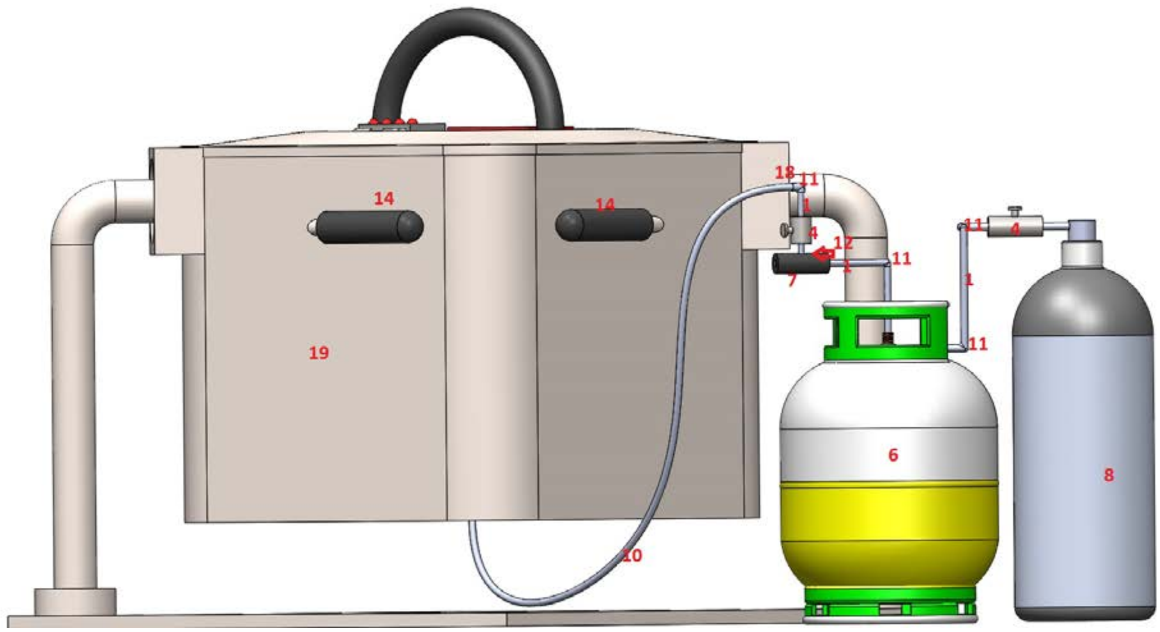

Figure 2: Assembly View 2

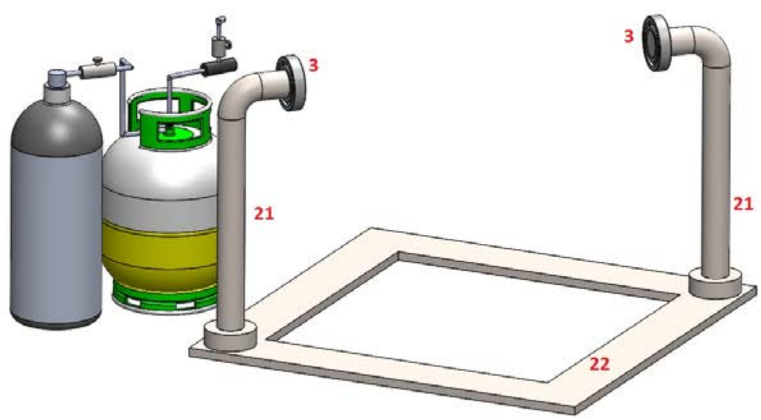

Figure 3: Subsystem View (supporting and rotation parts)

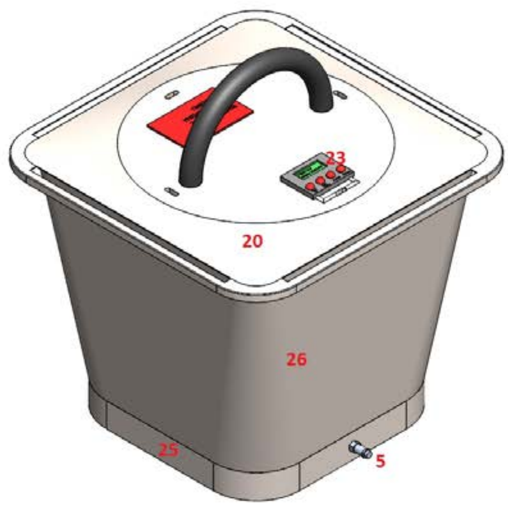

Figure 4: Isometric Subsystem View (w/o outer container)

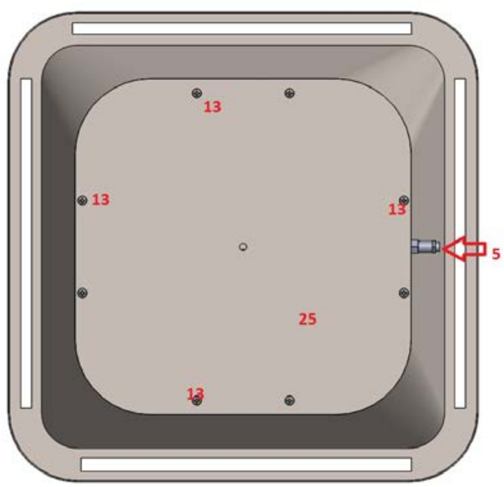

Figure 5: Bottom Subsystem View (w/o outer container)

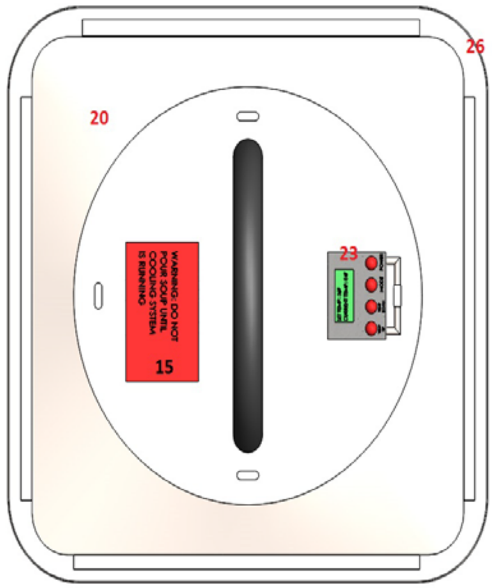

Figure 6: Top Subsystem View

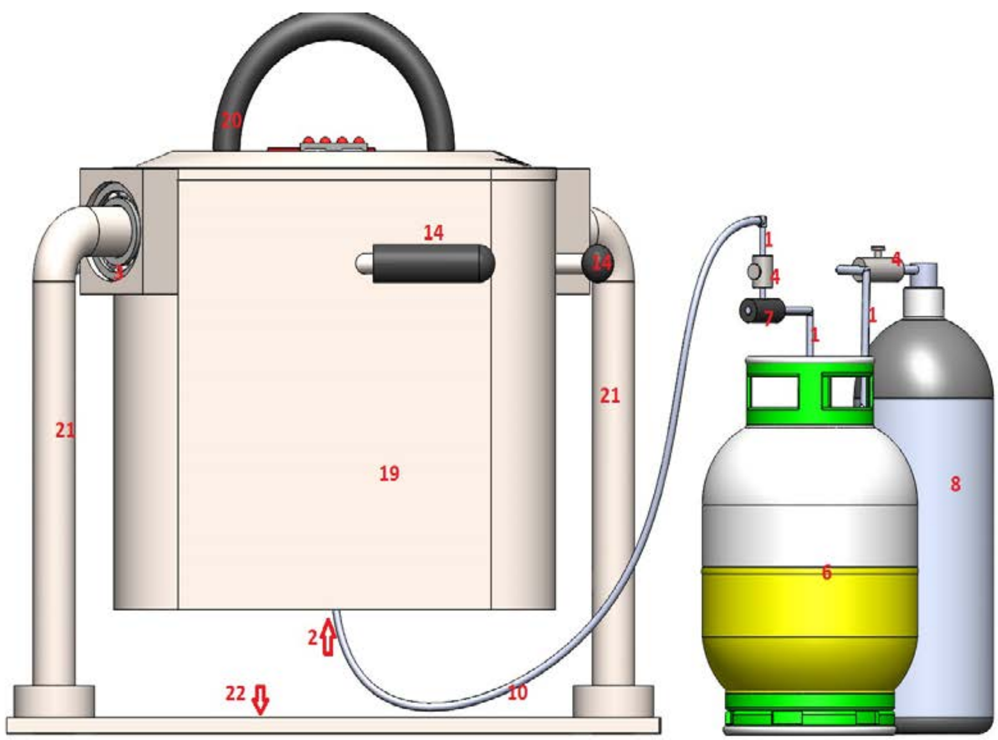

Figure 7: Front Assembly View

Table 1: Part Number and Corresponding Part

Part Number	Part
1	Type 304 Smooth-Bore Seamless Stainless Steel Tubing
1(A-H)	A: 2.5 inch SS piping B: 4 inch SS piping C: 1 inch capped SS piping D: SS piping feeding out of liquid N_2 E: SS piping feeding gaseous N_2 into liquid tank F: 4.5 inch SS piping G: 1.5 inch SS piping H: piping feeding out of Gaseous N_2
2	Precision AN 37°Flared Tube Fitting
3	Ball Bearings
4	Pressure Regulator
5	Adjustable Relief Valves
6	Liquid N_2 Container (CT-50)
7	Self-Regulating Heat Cable
8	Gaseous N_2 Container
9	N_2 Gas (working fluid)
10	Hose with Tube Connections for Liquid Nitrogen
11	Type 304 Stainless Steel Threaded Pipe L shaped Fitting
12	Type 304 Stainless Steel Threaded Pipe T shaped Fitting
13	Pan Head Phillips Machine Screws
14	Rubber handles

Table 1 Continued: Part Number and Corresponding Part	
15	Warning Sticker
16	Thermocouples (within inner container)
17	Electrical Wire (runs along piping and within outer container)
18	Type 316 Stainless Steel Yor-Lok Tube Fitting
19	Outer Container
20	Lid
21	L-bars for rotation
22	Square Base
23	LCD screen/control box
24	MicroController (inside control box)
25	Pressurized Chamber
26	Inner Soup Container
27	Micro USB cable to power MicroController
28	Buttons for MicroController

Assembly Instructions

- The Thermal system comes in four main sections. The first section is the lid, second is the inner container, the third section is the outer container and the fourth is the piping.

Lid:

No assembly necessary for the lid (20), it is one piece.The lid handle and surface are welded together and the controller box is wired and screwed through the lid.

Inner Container:

The inner section consist of two pieces, the pressurized chamber (25) and the inner soup container(26).

Outer Container:

This section is made of several parts including a large outer container (19), two bearings (3) press fitted into the outer container, L-shaped bars (21) press fitted into the bearings and a supportive base (22) welded to the end of the L-shaped bars. The outer container also has handles covered with rubber gripping.

Piping:

Piping is made up of several stainless steel pipes (1) connected by L-junctions (11) and a T-junction (12). These pipes run from the N_2 gas tank (8) to the Liquid N_2 tank (6), to the cooling system. There are also pressure valves (4) and a self-regulating cable heater (7) present in this section.

1. Using the eight stainless steel screws (13), screw the bottom piece of the inner container (26) into the pressurized chamber (25). There will be two screws on each side of the container.
2. Screw the relief valve (5) into the pressurized tank (25).
3. Place the inner container section (25 & 26) inside the outer container (19). Note that the inner container should not touch the bottom of the outer container, and will rest on the lip of the outer container.
4. The flexible stainless steel hose with liquid nitrogen connections (10) should be screwed to the precision AN 37°flared tube fitting (2).
5. Next, screw the other side of the precision AN 37°flared tube fitting (Part 2) onto the bottom of the pressurized chamber (25) which has a threaded hole.
6. Then screw the other end of the flexible stainless steel hose with liquid nitrogen connections (Part 10) onto the type 316 stainless steel yor-lok tube fitting (Part 18), which will allow it to connect to the rigid tubing made of type 304 smooth-bore seamless stainless steel(1A).
7. Connect the 2.5 inch rigid pipe(1A) to the bottom end of the T-junction (12), the 4 inch pipe (1B) to the right part of the junction and the 1 inch capped pipe (1C) to the other end. Attach an L-junction (11) to the end of the 4 inch pipe (1B) and then attach the other end of the junction to the pipe (1D) that is feeding out of the liquid nitrogen tank (6).
8. There is a horizontal pipe (1E) connected to the top of the liquid nitrogen tank (6), connect an L-junction to it and a 4.5 inch pipe (1F) to the other end. Attach another L-

junction to the top of that pipe and connect the 1.5 inch pipe (1G) to that junction. This pipe will connect to the pipe (1H) that is feeding out of the nitrogen gas tank (8).

9. Once piping is finished, wrap the self-regulating heater (7) around the pipe at the T-junction (12) and plug it into a nearby outlet.

Figure 8: Exploded view of the assembly

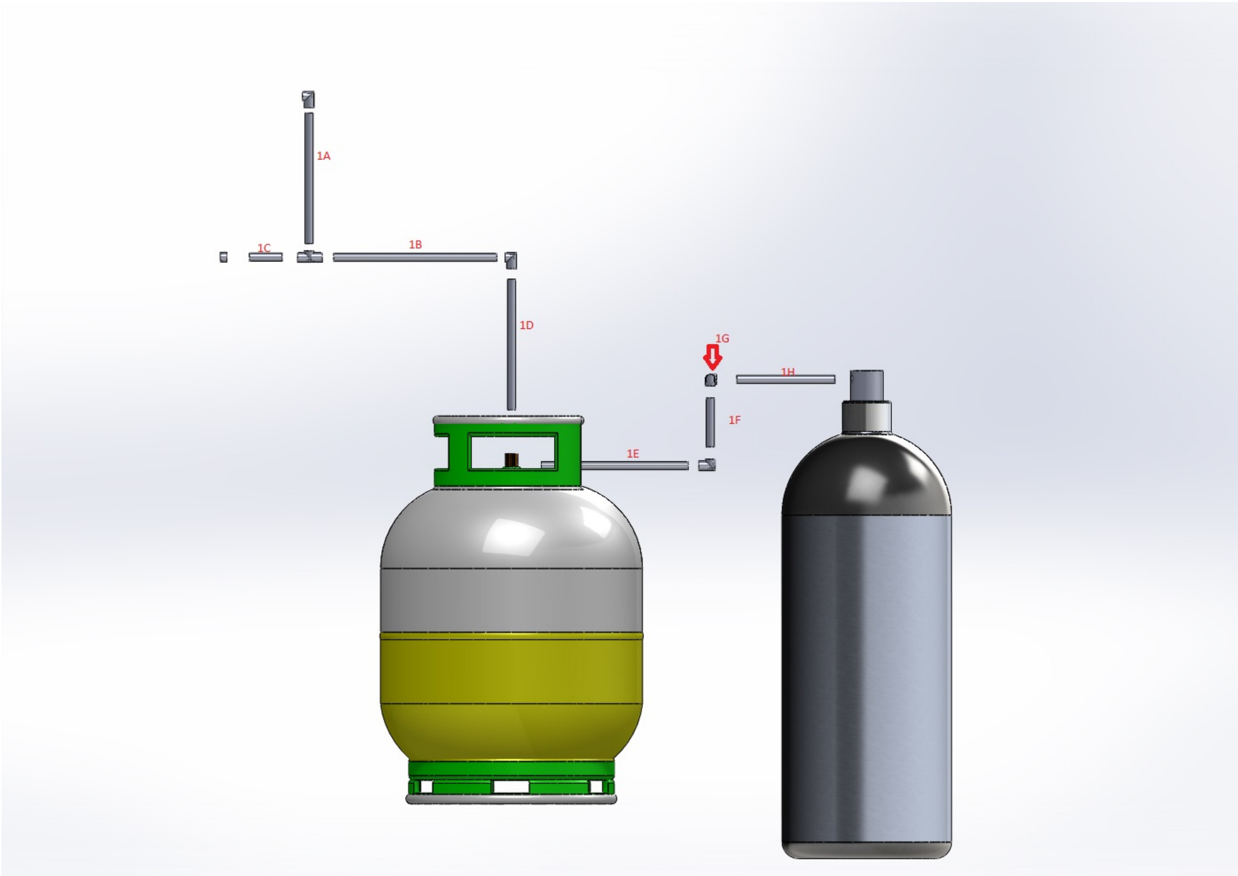

Figure 9: Pipe subassembly view

Cost Estimation:

See Appendix A for detailed cost analysis of individual components and the total estimated cost of the system prototype.

Detailed Thermal Analysis:

Schematic of Full System:

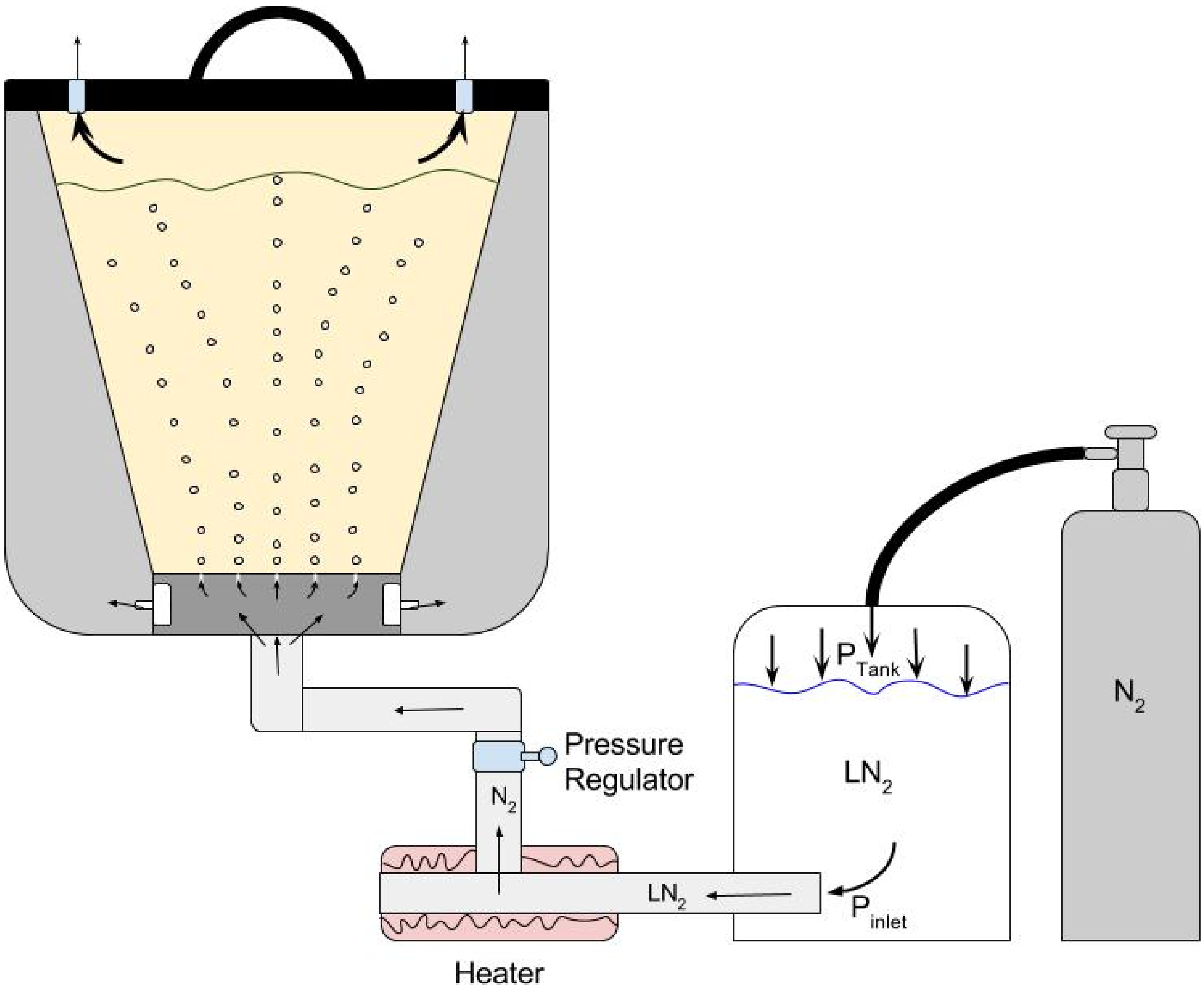

Figure 10: Schematic of Entire System

The above figure of the entire system is for visual and reference purposes only. For the ensuing thermodynamic analyses, the system will be broken up into many subsystems to be analyzed individually in order to solve for all necessary parameters.

Interior Soup Container Subsystem:

The thermal analysis begins by isolating the interior soup container where the primary form of heat transfer, between the injected gaseous nitrogen bubbles and the soup, occurs. Ultimately, the goal of analyzing this subsystem is to solve for the necessary mass flow rate of nitrogen gas in order to cool the soup from just below boiling point to a safe 40^{o}F in under four hours, as specified by the FDA. Below is a simplified schematic of the interior soup container to be used in the following analysis:

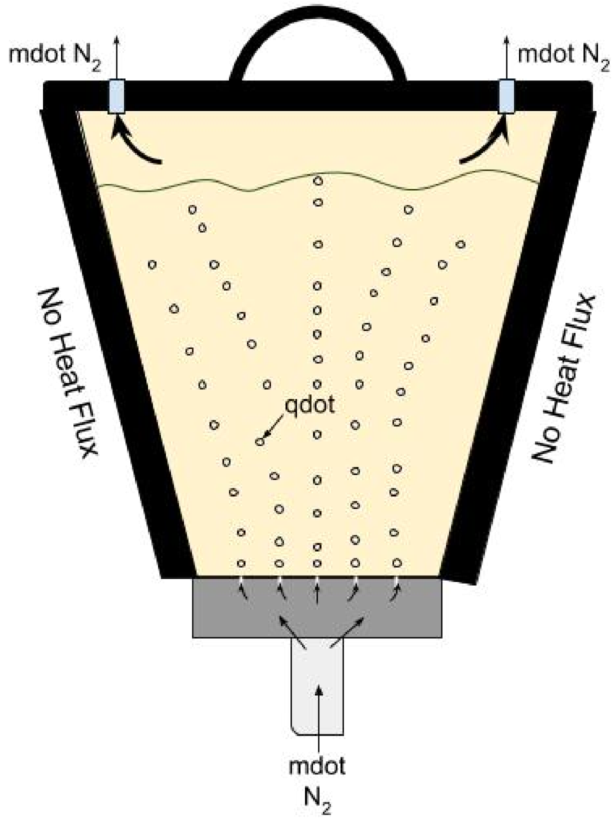

Figure 11: Interior Soup Container Subsystem

Assumptions:

The following assumptions apply for all thermal analyses of this subsystem

- The exterior walls are perfectly insulated so that there is no heat flux through the walls. Consequently, the only form of heat transfer is from the soup to the nitrogen gas bubbles. This assumption is used to model the worst case cooling scenario for gas injection.
- The flow of nitrogen gas pumped into the pressurized chamber below the interior container is equally forced through all of the small holes so that the mass flow rate through each hole is equivalent.
- The nitrogen bubbles reach thermal equilibrium with the soup by the time they reach the top level of the fluid. (This assumption will be investigated and validated through an analysis of the velocity of the nitrogen bubbles.)

- The soup will not freeze with an initial injected nitrogen gas temperature close to its boiling point. (This assumption will be investigated and validated through an energy balance of the soup close to its freezing point.)
- The specific heat of water was used for the specific heat of the soup and the density of an average chicken broth was used for the density of the soup. Both of these values were selected to model the worst case cooling scenario.

Governing Equations:

The following set of equations is used to determine the overall mass flow rate of gaseous nitrogen pumped through the system and into the soup.

Total Heat Loss Required From Soup:

$$Q_{total} = m_s c_{p,s}(T_{s,f} - T_{s,i}) \quad (1)$$

Where: m_s is the mass of the soup, $c_{p,s}$ is the specific heat of the soup, $T_{s,f}$ is the final temperature of the soup (40ºF), and $T_{s,i}$ is the initial temperature of the soup

Energy Balance Between Soup and Gas:

$$-\frac{m_s}{\Delta t} c_{p,s}(T_{s,f} - T_{s,i}) = \dot{m}_n c_{p,n}(T_{n,f} - T_{n,i}) \quad (2)$$

Where: $\dot{m}_n$ is the mass flow rate of the gas entering the system, $c_{p,n}$ is the specific heat of nitrogen gas, $T_{n,f}$ is the final temperature of the gas, and $T_{n,i}$ is the initial temperature of the soup

Difference Equation for Soup Temperature:

Using the the energy balance above (Equation 2) and the assumption that the final temperature of the nitrogen bubble is equal to the soup temperature, a difference equation can be set up to iteratively solve for the soup temperature after a small time step based on the previous soup temperature value.

$$T_{s,j} = \frac{m_s\, c_{p,s} T_{s,j-1} + \dot{m}_n\, c_{p,n}\, T_{n,i}\, \Delta t}{m_s\, c_{p,s} + \dot{m}_n\, c_{p,n}\, \Delta t} \quad (3)$$

This equation was used to create a simulation in MATLAB to model the soup temperature versus time where the mass flow rate was the main variable. Therefore, the simulation can be run for various nitrogen gas mass flow rates in order to determine a value that is best suited for the most efficient soup cooling.

The following set of equations is used to determine the velocity of the bubbles travelling through the soup in order to validate safe operating conditions.

Initial Velocity of Gas at the Small Hole Injection Sites:

$$v_{bubble} = \frac{\dot{m}_n}{n_h\ \rho_n\ A_{hole}}$$

(4)

Where: n_h is the number of holes (624), ρ_n density of nitrogen gas, A_{hole} is the area of the small injection hole

Equation of Motion for Nitrogen Bubble Through Soup:

$$\Sigma F = m_n\ a = F_B - F_g - F_D = \rho_s V_n g - m_n g - \frac{1}{2}\rho_s v_n^2 C_D A_n \quad (5)$$

Using the above force balance on a nitrogen gas bubble (Equation 5), a differential equation in terms of velocity can be formed and then integrated, given the initial velocity found from Equation 4, to solve for the bubble's velocity as a function of time.

$$\dot{v} = a = \frac{1}{m_n}\left(\rho_s V_n g - m_n g - \frac{1}{2}\rho_s v^2 C_D A_n\right)$$

(6)

Where: ρ_s is the density of the soup, V_n is the volume of a nitrogen gas bubble, v is the velocity of the bubble, C_D is the drag coefficient of a sphere, A_n is the cross sectional area of the bubble

The following equation is used to investigate and validate the assumption that the soup will not freeze for an initial injected nitrogen gas temperature close to its boiling point.

Energy Balance On Soup (Close to Freezing Point):

Given a mass flow rate solved for using Equations 1-3, an energy balance on the soup can be used to determine the time it takes for the soup to freeze. The energy balance equates the energy used to raise the temperature of the gas to the energy used to lower the soup temperature plus the energy required to change state from liquid to solid. As long as the system can cool the soup to the desired 40^{o}F in less than this freezing time, then this assumption holds and the soup will not freeze.

$$t = \frac{m_s c_{p,s}(T_{s,i} - T_{s,fp}) + m_s L_f}{\dot{m}_n c_{p,n}(T_{n,avg} - T_{n,i})}$$

(7)

Where: $T_{s,fp}$ is the freezing temperature of the soup, L_f is the latent heat of fusion of the soup

The following equation is used to determine the minimum pressure of the cavity below the internal soup container to ensure that no soup flows down into the small holes and into the cavity.

Pressure at the Bottom of Soup Container:

$$P_{min} = P_{atm} + \rho_s g h_{max}$$

(8)

Where: P_{min} is the minimum pressure required in the cavity, h_{max} is the maximum height of the soup (assumed to be the height of the interior soup container)

Numerical System Analysis:

Mass Flow Rate Calculation:

Using Equation 3 from above, a MATLAB simulation can be created that initially takes in the initial soup temperature and then iteratively calculates the next soup temperature after a short time step, Δt. The simulation takes in the following known values:

Known: $T_{s,i} = 373K, T_{n,i} = 82K, m_s = 21.3kg, c_{p,s} = 4179\frac{J}{kgK}, c_{p,n} = 1044\frac{J}{kgK}, \Delta t = 1s$

Find: The most efficient nitrogen gas mass flow rate to cool the soup in a maximum of 4 hours

Schematic: see Figure 9

Analysis:

Plugging in the known values, the difference equation (Equation 3) now becomes:

$$T_{s,j} = \frac{(21.3kg)(4179\frac{J}{kgK})T_{s,j-1} + \dot{m}_n (1044\frac{J}{kgK})(82K)(1s)}{(21.3kg)(4179\frac{J}{kgK}) + \dot{m}_n (1044\frac{J}{kgK})(1s)}$$

The MATLAB simulation was run for the prescribed 4 hours with mass flow rates of $\dot{m}_n = 0.005\frac{kg}{s}, 0.01\frac{kg}{s}, 0.015\frac{kg}{s}, 0.02\frac{kg}{s}$. The various mass flow rates created the following soup temperature functions versus time.

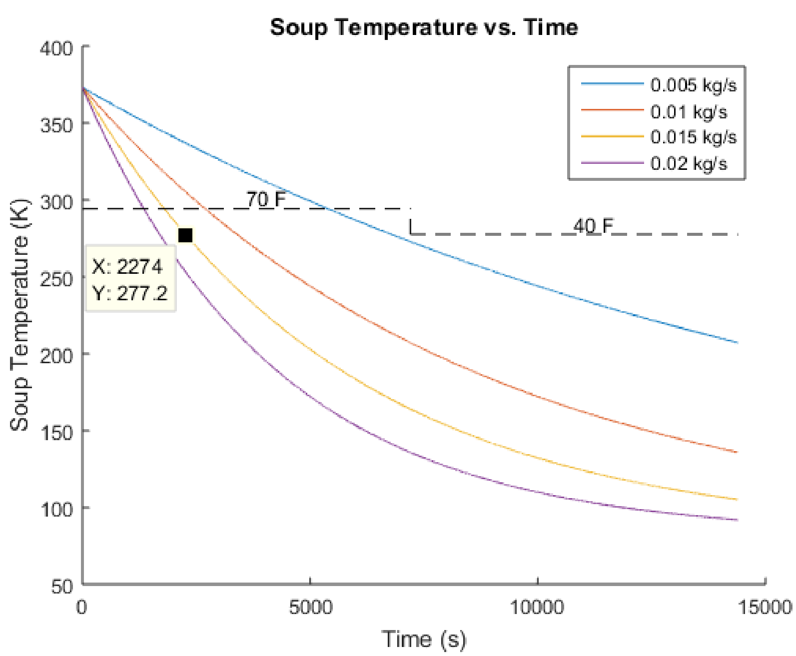

Figure 12: Soup Temperature vs. Time (Direct Injection Heat Transfer Only)

It can be seen from Figure 10 that as the mass flow rate increases, the time for the soup to reach 40ºF decreases. However, when considering the most efficient mass flow rate, it is important to note that a larger mass flow rate requires more liquid nitrogen which is a large operating cost. Therefore a mass flow rate of 0.015kg/s was determined to be the most efficient as it provides a relatively quick cooling time while minimizing the amount of liquid nitrogen required by the system.

Result: $\dot{m_n} = 0.015\frac{kg}{s}$ and cooling time, **t = 38 minutes**

Bubble Velocity Analysis:

In order to validate safe operating conditions, the position and velocity of a bubble of nitrogen gas versus time was solved for based on the equations of motion (Equations 5 and 6).

Known: $\rho_n = 4.6\frac{kg}{m^3}, \rho_s = 1065\frac{kg}{m^3}, d_n = 0.003m, V_n = 1.4 \times 10^{-8}m^3, A_n = 7.1 \times 10^{-6}m^2, C_D = 0.47$

Find: The velocity of the nitrogen bubbles as a function of time to ensure a safe speed

Schematic:

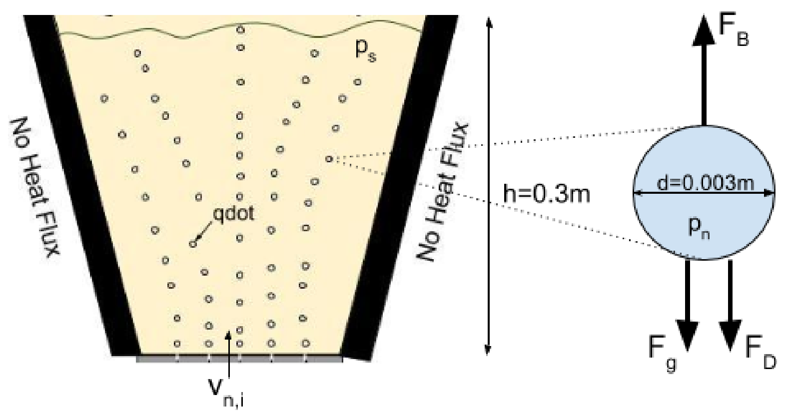

Figure 13: Schematic of Bubble's Motion and Free Body Diagram

Analysis:

Plugging in the known values into the differential equation for velocity (Equation 6) yields:

$$\dot{v} = \frac{1}{(4.6\frac{kg}{m^3})(1.4\times10^{-8}m^3)}[(1065\frac{kg}{m^3})(1.4 \times 10^{-8}m^3)(9.8\frac{m}{s^2}) - (4.6\frac{kg}{m^3})(1.4 \times 10^{-8}m^3)(9.8\frac{m}{s^2}) - \frac{1}{2}(1065\frac{kg}{m^3})v^2(0.47)(7.1 \times 10^{-6}m^2)]$$

This differential equation can now be solved using MATLAB's ode45 function, however, an initial velocity condition is required. This value can be solved for using Equation 4:

$$v_{bubble,i} = \frac{0.015\frac{kg}{s}}{(624\ holes)(4.6\frac{kg}{m^3})(7.1\times10^{-6}m^2)} = 0.74\frac{m}{s}$$

With this initial velocity condition, the following velocity and position functions were found:

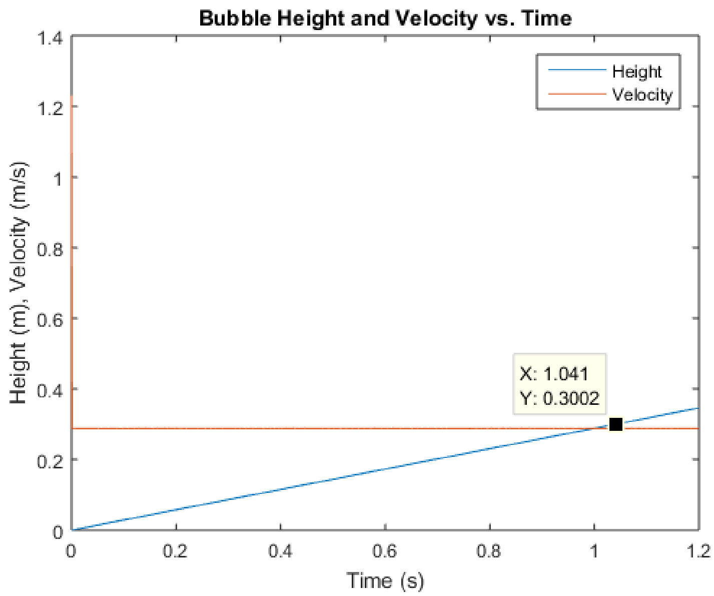

Figure 14: Bubble Height and Velocity vs. Time

Even with an initial velocity of 0.74m/s, the bubble quickly decelerates to a terminal velocity of about 0.29m/s.

__Results:__ Under these conditions, the bubble travels throughout the soup at a constant, terminal velocity of **0.29m/s** and takes a total of about **1.04 seconds** to rise through the soup. This velocity is fairly low in the sense that it certainly could not cause the soup to erupt out of the container or cause a jet stream of nitrogen to shoot out of the container and injury an operator.

Therefore, it can be concluded that a mass flow rate of 0.015 kg/s would cause completely **safe operating conditions**.

Potential Soup Freezing Analysis:

In order to validate the assumption that the soup will not freeze given an initial injected nitrogen gas temperature close to its boiling point, the maximum running time to avoid freezing can be found using Equation 7. Then this maximum running time can be compared to the proposed worst case running time found in the above mass flow rate analysis.

Known: $m_s = 21.3kg, c_{p,s} = 4179\frac{J}{kgK}, c_{p,n} = 1044\frac{J}{kgK}, T_{s,i} = 373K, T_{s,fp} = 273K$

$$\dot{m}_n = 0.015\frac{kg}{s}, T_{n,i} = 82K, T_{n,avg} = 323K, L_f = 333.6\frac{kJ}{kg}$$

Find: The maximum running time to avoid freezing

Schematic: see Figure 9

Analysis:

Plugging in the known values into Equation 7 yields the following:

$$t = \frac{(21.3kg)(4179\frac{J}{kgK})(373K - 273K) + (21.3kg)(333{,}600\frac{J}{kg})}{(0.015)(1044\frac{J}{kgK})(323K - 82K)}$$

$$= 4241s = 71min$$

Results: The maximum running time to avoid freezing is 71 minutes. Since the derived worst case running time for a mass flow rate of 0.015kg/s was about 38 minutes, the assumption that the soup does not freeze is validated.

Minimum Operating Pressure Analysis:

In order to solve for the minimum operating pressure in the cavity below the interior soup container, the known thermodynamic and system properties can be inserted into Equation 8.

Known: $\rho_s = 1065 \frac{kg}{m^3}, h_{max} = 0.3m$

Find: The minimum operating pressure in the cavity below the interior soup container to avoid backflow

Schematic: see Figure 9

Analysis:

Plugging in the known values into Equation 8 yields the following:

$$P_{min} = 101000\ Pa + (1065 \frac{kg}{m^3})(9.8 \frac{m}{s^2})(0.3m) = 104131\ Pa$$
$$= 15.10\ psi$$

Results: The minimum operating pressure inside the cavity below the interior soup container is an absolute pressure of **104131 Pa or 15.10 psi**.

Interior Wall Conduction Subsystem:

The purpose of this subsystem was to determine the effectiveness of cooling the exterior of the internal soup container with nitrogen gas. By isolating the subsystem, we were able to model how heat leaves the system through the walls for various mass flow rates. The cooling method represented in the schematic below will be in addition to the main cooling subsystem discussed above.

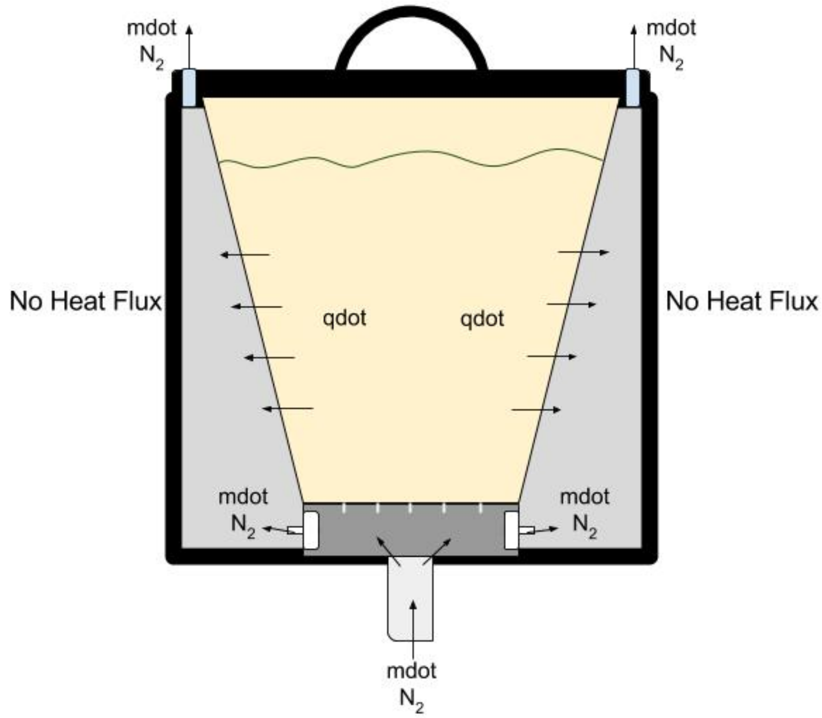

Figure 15: Interior wall conduction subsystem

Assumptions:

The following assumptions were used throughout the analysis of this subsystem.

- All heat from the soup is transferred to the gas in the pocket outside the interior container
- One dimensional heat transfer through the interior wall
- The temperature of the gas is constant throughout the pocket
- The specific heat of water was used for the specific heat of the soup and the density of an average chicken broth was used for the density of the soup. Both of these values were selected to model the worst case cooling scenario.

Governing Equations:

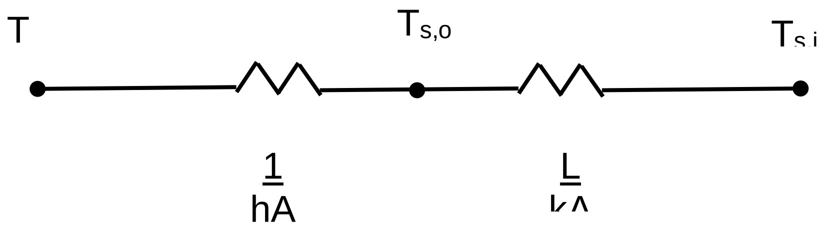

Figure 16: Interior wall thermal resistance network

The thermal resistance network representing the gas and the inside of the interior wall is presented above. Using the network, an equation can be derived for the heat transfer between the gas and the inside of the soup container.

$$q = \frac{T_{s,i} - T_n}{\frac{1}{hA} + \frac{L}{kA}} = \frac{T_{s,i} - T_n}{R_{tot}} \qquad (9)$$

Where q is the heat rate, $T_{s,i}$ is the temperature of the inside wall of the internal container, T_n is the temperature of the nitrogen, h is the convection coefficient, A is the surface area, L is the thickness of the wall, k is the thermal conductivity of the wall, and R_{tot} is the total resistance between the nitrogen and the inside wall as shown in equation 10.

$$R_{tot} = \frac{1}{hA} + \frac{L}{kA} \qquad (10)$$

<u>Convection coefficient calculation:</u>

The convection coefficient was calculated using a Nusselt number correlation (8.60) found in *Fundamentals of Mass and Heat Transfer*. Due to the change in diameter along the length of the soup cooler, the Reynolds number was calculated using the average hydraulic diameter found at the midpoint.

$$Re_D = \frac{4\dot{m}_n}{\pi \mu D_H} \qquad (11)$$

Where $\dot{m}_n$ is the mass flow rate of the nitrogen, and D_H is the hydraulic diameter

$$Nu_D = 0.023 Re_D^{4/5} Pr^{0.4} \qquad (12)$$

$$h = \frac{Nu_D k}{D_H} \qquad (13)$$

<u>Energy Balance between the gas and the soup</u>

By introducing a time constant into equation 9, it can be set equal to equation 1 found in the first subsystem assuming the temperature of the wall is the same as the soup, the result is equation 14.

$$\frac{(T_s - T_n)}{R_{tot}} = -\frac{m_s}{\Delta t} c_{p,s}(T_s - T_{s,i}) \qquad (14)$$

Where T_s is the temperature of the soup that needs to be solved for, and $T_{s,i}$ is the temperature of the soup at the previous time step that's known.

Solving the equation for T_s yields equation 15, which can be used to find the temperature of the soup after a time step of Δt.

$$T_{s,j} = \frac{T_n \Delta t + m_s c_{p,s} R_t T_{s,j-1}}{\Delta t + m_s c_{p,s} R_t} \quad (15)$$

Using equation 14, the temperature distribution of the soup can be found numerically, as can the time required for the soup to reach the designated temperatures.

Numerical System Analysis

Mass flow rate calculation

Known: $T_{s,i} = 373K, T_{n,i} = 82K, m_s = 21.3kg, c_{p,s} = 4179 \frac{J}{kgK}, c_{p,n} = 1044 \frac{J}{kgK}, \Delta t = 1s$

$$Pr = 0.7215, k_{steel} = 15\frac{W}{m \cdot K}, k_n = 0.0241\frac{W}{m \cdot K}, D_H = 0.0976m, A_s = 0.373m, L = 0.0012m$$

$$\mu = 1.67 \times 10^{-5}\ Pa \cdot s$$

Find:

a) The minimum mass flow rate necessary to cool the soup within four hours
b) Compare to the mass flow rate found in subsystem 1

Analysis:

Plugging in the known values starting from equation 10 for a mass flow rate of $\dot{m}_n = 0.015\frac{kg}{s}$

$$Re_D = \frac{4(0.015\frac{kg}{s})}{\pi(1.67 \times 10^{-5}\ Pa \cdot s)(0.0976m)} = 11{,}700$$

$$Nu_D = 0.023(11{,}700)^{4/5}(0.7215)^{0.4} = 36.4$$

$$h = \frac{(36.4)(0.0241\frac{W}{m \cdot K})}{0.0976m} = 8.98\frac{W}{m^2 \cdot K}$$

$$R_{tot} = \frac{1}{(8.98\frac{W}{m^2 \cdot K})(0.373m)} + \frac{0.0012m}{(15\frac{W}{m \cdot K})(0.373m)} = 0.299\frac{m \cdot K}{W}$$

$$T_{s,j} = \frac{(82K)(1s) + (21.3kg)(4179\frac{J}{kgK})(0.299\frac{m \cdot K}{W})T_{s,j-1}}{(1s) + (21.3kg)(4179\frac{J}{kgK})(0.299\frac{m \cdot K}{W})}$$

Using MATLAB, temperature versus time plots were graphed to find the minimum mass flow rate required to meet the cooling conditions.

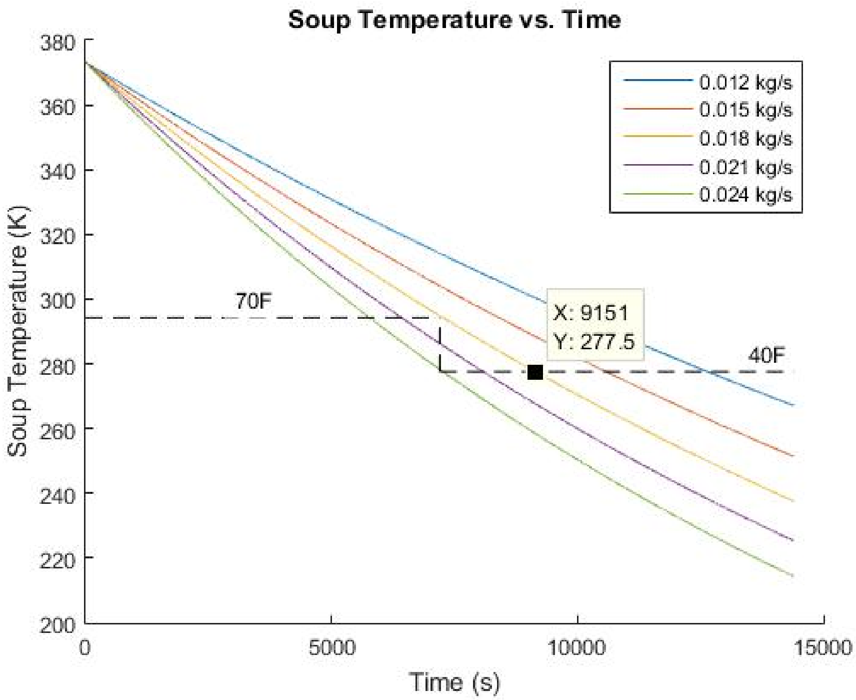

Figure 17: Soup Temperature vs Time only using surface cooling

Results:

The minimum mass flow rate was $\dot{m}_n =$ **0.018**$\frac{kg}{s}$ with a cooling time of t = **2.54 hours**. The analysis showed that it's possible to cool the soup using only surface cooling, however, the mass flow rate would have to increase by 20%. The cooling time is also significantly longer which will use more nitrogen than the injection method. The data suggests that the method is inefficient when compared to direct injection, and upon combining the two methods, we found that our system was optimized when using only direct injection. So the final design doesn't support surface cooling through the walls with nitrogen.

Direct Injection with Interior Wall Conduction Subsystem:

In this subsystem, the two previous forms of thermal analysis will be combined to model the heat transfer out of the soup and into the injected nitrogen bubbles as well as out of the interior wall through conduction. It has already been shown that the direct injection method is the most efficient form of heat removal, therefore, the soup will cool at the quickest rate when all of the 0.015 kg/s mass flow rate is injected directly into the soup and none of the gas is injected in the space between the interior and exterior containers. As a result, the primary form of heat transfer will be through direct injection, but there is no longer the assumption that there is no heat flux through the interior walls. Therefore, the thermal analysis from the previous section (Interior Wall Conduction Subsystem) is still useful in this subsystem because, although there is no gas injected in the side space, room temperature air now acts as a convective fluid to help conduct heat through the interior wall and promote additional heat removal out of the soup. The intention is to produce the most realistic model for this system in order to calculate the concrete running time of the soup cooler.

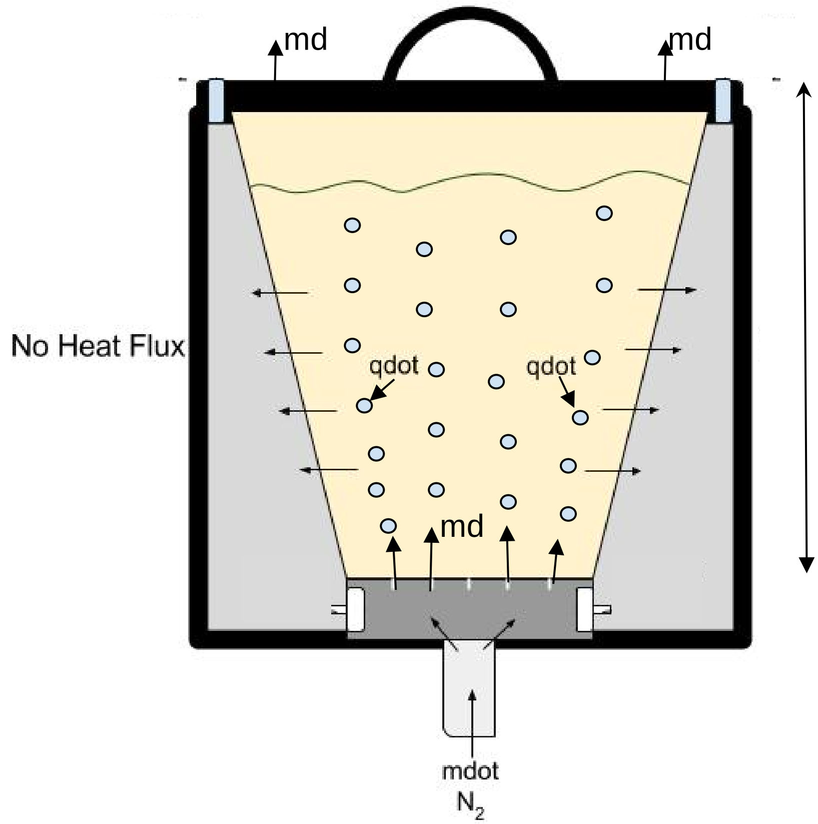

Figure 18: Direct Injection with Interior Wall Conduction Subsystem

Assumptions:

- Air in the space between the interior and exterior walls is kept constant at room temperature.
- Conduction through wall can be modelled as one-dimensional heat transfer
- The flow of nitrogen gas pumped into the pressurized chamber below the interior container is equally forced through all of the small holes so that the mass flow rate through each hole is equivalent.
- The nitrogen bubbles reach thermal equilibrium with the soup by the time they reach the top level of the fluid. (Validated in Interior Soup Container Subsystem)
- The soup will not freeze with an initial injected nitrogen gas temperature close to its boiling point. (Validated in Interior Soup Container Subsystem)
- The specific heat of water was used for the specific heat of the soup and the density of an average chicken broth was used for the density of the soup. Both of these values were selected to model the worst case cooling scenario.

Governing Equations:

The following set of equations is used to develop a MATLAB simulation to model the temperature of the soup versus time considering both forms of heat transfer as previously discussed. From this MATLAB simulation and the predetermined mass flow rate of nitrogen gas, the total running time can then be found.

Overall Energy Balance:

$$\dot{Q}_{soup} = \dot{Q}_{cond} + \dot{Q}_{gi} \tag{16}$$

Solving for the individual heat rates above yields the following in depth energy balance:

$$\frac{-m_s c_{p,s}(T_{f,s}-T_{i,s})}{\Delta t} = \frac{(T_{i,s}-T_{i,n})}{R_{tot}} + \dot{m}_n c_{p,n}(T_{f,s} - T_{i,n})$$

(17)

Using the the energy balance above (Equation 17) and the assumption that the final temperature of the nitrogen bubble is equal to the soup temperature, a difference equation can be set up to iteratively solve for the soup temperature after a small time step based on the previous soup temperature value.

$$T_{j,s} = \frac{\frac{1}{R_{tot}}\Delta t(T_{j-1,s}-T_{i,n}) - \dot{m}_n c_{p,n}\,\Delta t\, T_{i,n} - m_s c_{p,s} T_{j-1,s}}{(-m_s c_{p,s} - \dot{m}_n c_{p,n}\Delta t)}$$

(18)

Numerical System Analysis:

Running Time Calculation:

Using Equation 18 from above, a MATLAB simulation can be created that initially takes in the initial soup temperature and then iteratively calculates the next soup temperature after a short time step, Δt. The simulation takes in the following known values:

Known: $T_{s,i} = 373K, T_{n,i} = 82K, m_s = 21.3kg, c_{p,s} = 4179\frac{J}{kgK}, c_{p,n} = 1044\frac{J}{kgK}, \Delta t = 1s$

$$k_{steel} = 15\frac{W}{m \cdot K}, A_s = 0.373m, L = 0.0012m, h_{air} = 10\frac{W}{m^2K}, \dot{m}_n = 0.015\frac{kg}{s}$$

Find: The total running time for the system to cool the soup to 40°F given a gaseous nitrogen mass flow rate of $0.015\frac{kg}{s}$.

__Schematic:__ see Figure 17

__Analysis:__

Plugging in the known values into Equation 10 for the thermal resistance and Equation 18 for the overall difference equation yields the following:

$$R_{tot} = \frac{1}{\left(10\,\frac{W}{m^2K}\right)(0.373m)} + \frac{0.0012m}{\left(15\,\frac{W}{m \cdot K}\right)(0.373m)} = 0.27\,\frac{m \cdot K}{W}$$

$$T_{j,s} = \frac{\frac{1}{\left(0.27\,\frac{m \cdot K}{W}\right)}(1s)(T_{j-1,s} - 82K) - \left(0.015\,\frac{kg}{s}\right)\left(1044\,\frac{J}{kgK}\right)(1s)(82K)}{-(21.3kg)\left(4179\,\frac{J}{kgK}\right) - \left(0.015\,\frac{kg}{s}\right)(1044 \ldots}$$

The MATLAB simulation can now be run for the given mass flow rate which produces a temperature function as shown in the following figure.

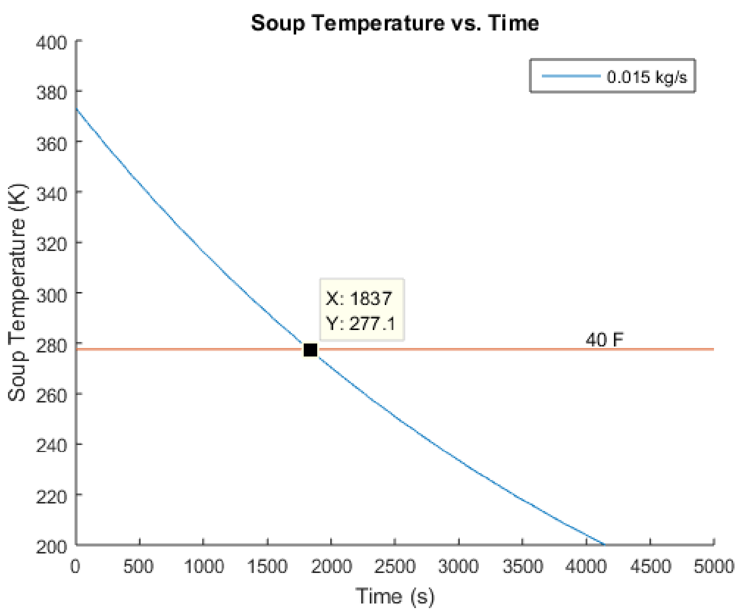

Figure 19: Soup Temperature vs. Time with Direct Injection and Surface Cooling

__Results:__

It can be seen from Figure 18 that it will take a total of about 1837 seconds for the system to cool the soup to the desired 40ºF temperature with a nitrogen gas mass flow rate of 0.015kg/s.

Total Running Time = 1837 sec = 30 minutes

Heater and Pressure Regulator subsystem:

The heater and pressure regulator are two necessary components to control the temperature and amount of gaseous nitrogen injected into the system. The required power and pressure from the heater and regulator will be used in conjunction with a thermocouple in the soup as the three main components of the control system for our soup cooler. A quick note on a large assumption made in this analysis is that the liquid nitrogen is assumed to transfer to a gaseous stage during its travel through the stainless steel tubing before it enters the heater. One reason being that the power required from the heater to vaporize liquid nitrogen is on the verge of kilowatts which is unrealistic for a tube resistance heater. The other reason being the commonly experienced phenomena seen when liquid nitrogen is poured out of a container and vaporizes before it even contacts the ground due to the temperature of ambient air. We assume this same phenomena will occur when the liquid nitrogen is passed through the tubing at ambient temperature, and the heater operating at reasonable power will assure the vaporization occurs by adding the necessary energy the liquid. The outlet of the heater will also be vertical to assure that only gaseous nitrogen passes through the system and all remaining liquid nitrogen will remain at the base of the heater until enough energy is transferred to it to change phase and pass through the vertical portion as well.

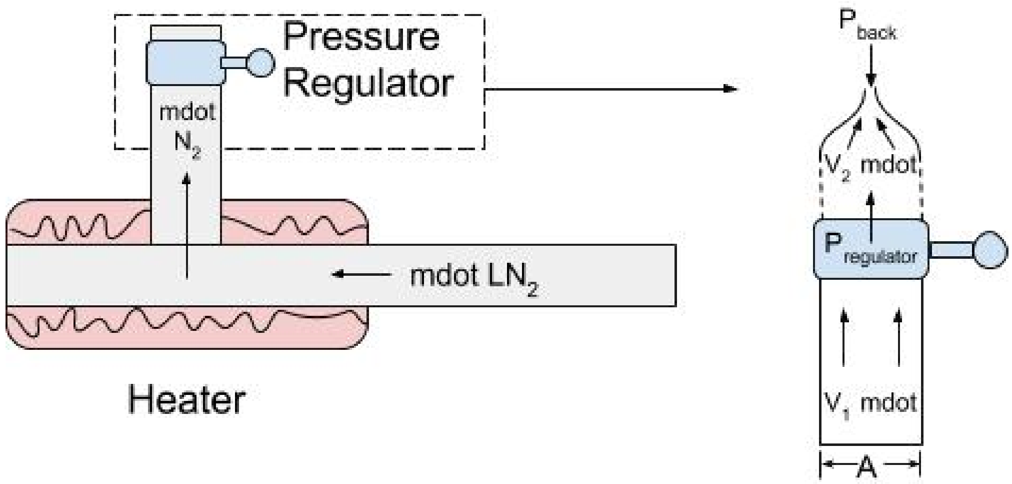

Figure 20: Heater and Pressure Regulator Subsystem

Assumptions:

- The heater is perfectly efficient
- No head loss through pipes ($\varepsilon = 0$) meaning the mass flow rate is constant
- The liquid nitrogen vaporizes as it is exposed to tubing at ambient temperature, therefore it is not necessary to provide enough power through the heater to overcome the latent heat of vaporization for nitrogen
- There is no significant height difference between the outlet of the pressure regulator and the entrance of the gas bubbles into the soup

Governing Equations

The heater has two simple equations to regulate the desired voltage input to receive a desired temperature output. Those equations are first the definition of electrical power:

$$P = VI \tag{19}$$

The voltage will be changed as necessary to reach a desired power output based on the rate of heat transfer $\dot{Q}$:

$$P = \dot{Q} \; = \dot{m} c_p (T_{outlet} - T_{inlet}) \tag{20}$$

The mass flow rate $\dot{m}$ is the same constant flow rate throughout the system and c_p is the specific heat of gaseous nitrogen. The inlet temperature will remain at a constant value equal to the boiling point of liquid nitrogen (-196∘C) because the gaseous nitrogen will just have vaporized after traveling through the pipes at ambient temperature. The T_{outlet} is what we control and is what changes based on the power input. This temperature is a value we determine through analysis is the soup so we can solve for the desired power output of the heater.

The governing equations for the pressure regulator are based on a Bernoulli equation between the outlet of the heater and the inlet of one hole of the soup pot.

$$P_2 + \frac{1}{2}\rho V_2^2 + \rho g h_2 = P_1 + \frac{1}{2}\rho V_1^2 + \rho g h_1 \tag{21}$$

The heights between these two points are assumed to be the same therefore the Bernoulli equation reduces to

$$P_2 + \frac{1}{2}\rho V_2^2 = P_1 + \frac{1}{2}\rho V_1^2 \tag{22}$$

P_2 was found in the interior soup container subsystem and is dependent on the buoyant force of the soup and the atmospheric pressure. We must still find values for both velocities to be able to solve for pressure of the regulator P_1, and that is done as follows:

$$V_{1,2} = \frac{\dot{m}}{\rho_{N_2} A_c} \tag{23}$$

Both values are needed to evaluate the bernoulli equation and are products of $\dot{m}$, the mass flow rate of the system divided by the density of the gas ρN_2 and A_c the cross sectional area of the pipe at each point.

Numerical Heater Subsystem Analysis

<u>Knowns:</u>

$\dot{m} = .015\frac{kg}{s}$ $c_p = 1039\frac{J}{kgK}$ T_{inlet} = -196∘C T_{outlet} = -191∘C

<u>Find:</u>

Power required from heater

<u>Schematic:</u>

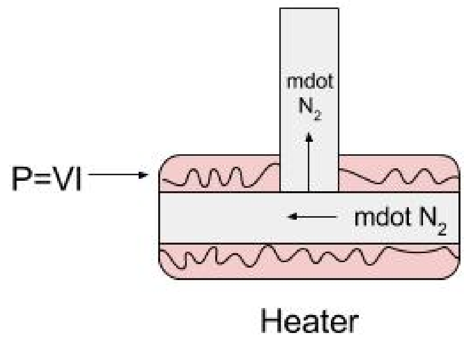

Figure 21: Schematic of Heater Subsystem

Analysis:

$P = \dot{Q} = \dot{m}c_p(T_{outlet} - T_{inlet})= .015\frac{kg}{s} * 1039\frac{J}{kgK}(-191 \circ C - 196 \circ C)$ = **77.925 Watts**

Results:

The constant power required from the heater assuming the input temperature remains at the boiling point of nitrogen is **77.925 Watts**

Numerical Pressure Regulator Subsystem:

Knowns: $\rho_{N_2} = 4.6\frac{kg}{m^3}$ P_2=104131 Pa

Find:

The velocity at the pressure regulator and at the soup inlet

The Pressure output required from the pressure regulator $P_{regulator}$

Schematic:

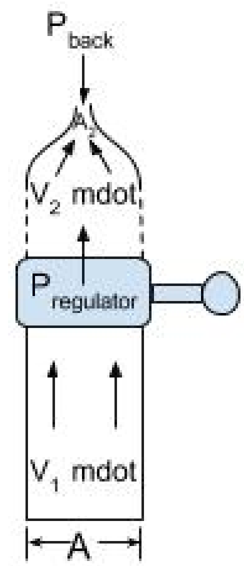

Figure 22: Schematic of Pressure Regulator Subsystem

Analysis:

$$V_1 = \frac{\dot{m}}{\rho_{N_2} A_1} = \frac{.015\frac{kg}{s}}{4.6\frac{kg}{m^3} * \pi * (0.00493m)^2/4} = 170.82 \text{ m/s}$$

$$V_2 = \frac{\dot{m}}{n_{holes}\rho_{N_2} A_2} = \frac{.015\frac{kg}{s}}{4.6\frac{kg}{m^3} * \pi * (0.003m)^2/4*624} = .739 \text{ m/s}$$

$$P_{regulator} = P_2 + \frac{1}{2}\rho V_2^2 - \frac{1}{2}\rho V_1^2 = 104131\ Pa\ + \frac{1}{2} 4.6 \frac{kg}{m^3} * ((.739\frac{m}{s})^2 - (170\frac{m}{s})^2)$$

$$P_{regulator} = \mathbf{37662.26\ Pa = 5.46\ psi}$$

Results:

The required absolute pressure for the regulator to output is only **37662.26 Pa or 5.46 psi.** This is assuming that the pot of soup is filled to the maximum, the user will input the height of the soup into the system controller based on markings inside the pot, and this new height will change the input to the Pressure regulator based on the difference in the P_2 value.

Tank Pressure Subsystem:

The pressure from the tank is controlled to ensure a constant initial mass flow rate of liquid nitrogen through the system and to ensure that the pressure from the the outlet of the bubbles is enough to resist the buoyant force of the soup. Through the analysis of this subsystem along with the required pressure from the tank, the values of required liquid nitrogen and gaseous nitrogen for each run will be evaluated.

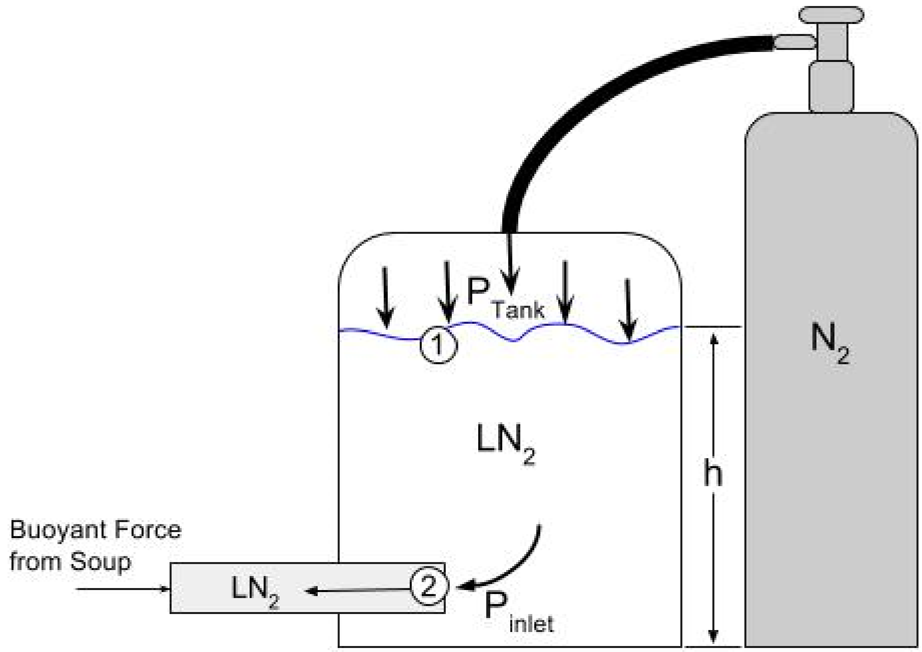

Figure 23: Tank Pressure Subsystem

Assumptions:

- No minor or major head loss at elbow connection of tube and tank and throughout the tubes.
- Liquid Nitrogen is an incompressible fluid
- Density and Volume of LIN are conserved

Governing Equations:

The following calculations were used to determine the pressure required from the N_2 tank to push the proper amount of liquid nitrogen through the system, as well as calculate the required tank sizes and amounts of nitrogen.

<u>Velocity at location 2:</u>

$$V_2 = \frac{\dot{m}}{\rho_{LIN} A_c} \tag{24}$$

The velocity at the outlet V_2 is needed to evaluate the bernoulli equation to follow and is a product of $\dot{m}$, the mass flow rate of the system divided by the density of the liquid ρ_{LIN} and A_c the cross sectional area of the pipe at that point.

<u>Bernoulli Equation relating locations 1 and 2:</u>

$$P_2 + \frac{1}{2}\rho V_2^2 + \rho g h_2 = P_1 + \frac{1}{2}\rho V_1^2 + \rho g h_1 \tag{25}$$

The bernoulli equation here is used to evaluate the pressure. A few variables in this equation will fortunately cancel out such as the height at location 2 (ground level) and the velocity at location 1 (Volume of tank so large that height doesn't vary with time.) P_2 is calculated in subsystem 1 as the combined buoyant pressure from the soup in the pot and the atmospheric pressure. P_1 is what we are solving for and is the pressure from the gaseous nitrogen tank.

Liquid Nitrogen Required in tank:

$$V = \frac{\dot{m}t_{total}}{\rho_{LIN}} \tag{26}$$

The volume of liquid nitrogen required to cool the soup is the mass flow rate times the total time it takes to cool the system, found in the direct injection with interior wall conduction subsystem, divided by the density of liquid nitrogen.

Gaseous Nitrogen Required in tank:

The volumetric flow rate of the release of liquid nitrogen into the system is the same as the volumetric flow rate of gaseous nitrogen into the tank, therefore the required amount of gaseous nitrogen will be the same as liquid nitrogen and again be determined by equation (26) and the total time it takes to cool the soup.

Numerical Tank Pressure Subsystem:

Known:

P_2 = 104131 Pa $\rho_{LIN} = 807\frac{kg}{m^3}$ $g = 9.81\frac{m}{s^2}$ $h_1 = .23m$ $t_{total} = 30\ min$ $\dot{m} = .015\frac{kg}{s}$

Find:

Pressure of the gaseous nitrogen tank

Total amount of liquid and gaseous nitrogen consumed throughout the cooling process

Schematic:

View Figure 21

Analysis:

$$V_2 = \frac{\dot{m}}{\rho_{LIN}\ \ A_2} = \frac{.015\frac{kg}{s}}{807\frac{kg}{m^3}*\pi*(0.00493m)^2/4} = .974\ \text{m/s}$$

$$P_2 + \frac{1}{2}\rho_{LIN}V_2^2 = P_1 + \rho_{LIN}gh_1$$

$$P_1 = 104131\ Pa + .5 * 807\frac{kg}{m^3} * (.974\frac{m}{s})^2 - 807\frac{kg}{m^3} * 9.81\frac{m}{s^2} * .23m$$

P_1= **102693 Pa = 14.89 psi**

$$V = \frac{\dot{m}t_{total}}{\rho_{LIN}} = \frac{.015\frac{kg}{s}*30min*60sec*1000L}{807\frac{kg}{m^3}*1min*1m^3} = \mathbf{33.46\ L}$$

<u>Results:</u>

The total pressure required from the tank is **102693 Pa** or **14.89 psi**, this is only a shade over atmospheric pressure, therefore finding a tank to release at this pressure should not be difficult. The volume of liquid and gaseous nitrogen required for each cooling cycle is **33.46 L,** this value means that after every soup is cooled, the tank of liquid nitrogen and gaseous nitrogen would most likely need to be refilled.

<u>Control System Overview</u>

The following is an overview of the control system and its key components.

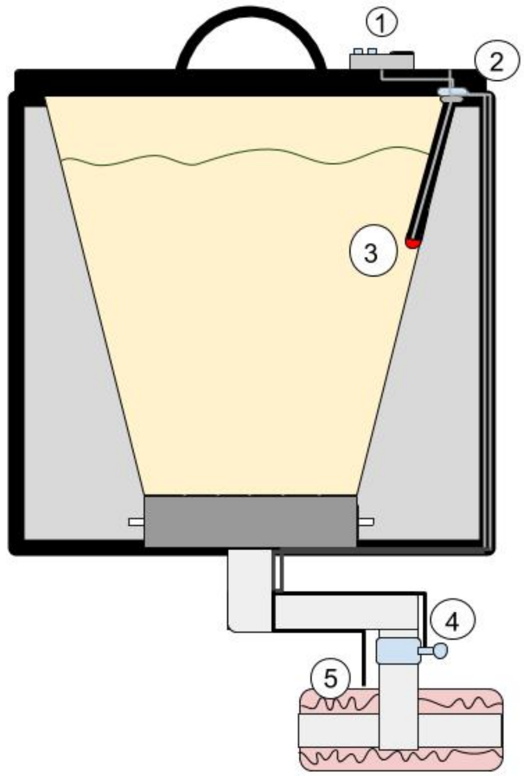

Figure 24: Control System Overview

The control system is essential for measuring the current and desired temperature of the system and utilizing the thermal analysis equations to handle the pressure regulator and heater output. The T-type thermocouple at location 3 in Figure 24 will display the temperature on the control panel at location 1. The control panel will be attached to the top of the lid and the wiring connected to it will run through the outside edge of the lid until the wiring is attached to a conductive metal strip with insulated edges that is taped to the bottom of the lid. This piece will be placed flush on another conductive metal strip taped to the top of the lid that is connected to the wires that pass through the edge of the tank to the pressure regulator and heater. This connection is seen at location 2 in figure 23 and the control panel will light up when the user has the lid placed in the right orientation. The user will input the desired end temperature of the soup and once the thermocouple reaches that temperature the control system will communicate with the pressure regulator at location 4 to only output enough gas to prevent soup from entering through the holes in the tank (calculated in Eq. 8) and release the rest to the atmosphere. The control panel will then signal to the user through a constant beeping that the soup is cooled and the gaseous nitrogen tank may be closed and the heater, location 5, may be removed from its source.

The control system will utilize a handful of the thermal analysis equations that have been discussed in previous subsections to mitigate the error at each time step and deliver the desired output. Ultimately, it has been shown through the previous thermal analysis that the main variables that affect the soup temperature and cooling time are the mass flow rate of nitrogen gas and the temperature of this nitrogen gas that is injected into the soup.

Therefore, in order to control the mass flow rate, the controller must be programmed using the Bernoulli flow analysis from equations 22 and 23. Because the controller will be able to electronically tune the pressure regulator after the heater, it will be able to calculate the pressure necessary to achieve a velocity and thus mass flow rate of nitrogen gas to cool the soup to the desired, set temperature.

Additionally, the temperature of the nitrogen gas that enters the soup container must be controlled in order to maintain safe, functional operation and to cool the soup in the necessary time. The controller will be able to tune the nitrogen temperature by applying variable voltage to the electric heater. Therefore, the controller will need to be programmed using the energy conservation equation at the heater (Equation 20). Using this as a fundamental equation, the controller will be able to apply the necessary voltage to control the power applied to raise the temperature of the gas.

Now with fundamental equations that describe how to tune the main two input variables (mass flow rate and temperature of the gas), the controller can then use these values to simulate the overall running time required to cool the soup to the desired temperature as inputted by the user in a given amount of time. In order to create such a simulation, the controller must utilize the overall difference equation (Equation 18) to calculate the overall running time. With all of the above information, the controller now can tune the flow rate and nitrogen temperature to achieve any final soup temperature and running time as specified by the user.

Appendix A

Off the shelf components

Table 2 : Appendix A- Off the shelf components

#	Component	Url	Quantity	Price
1	Type 304 Smooth-Bore Seamless Stainless Steel Tubing	Stainless Steel Tubing	1 (3ft)	$16.98
2	Precision AN 37°Flared Tube Fitting	Tube Fitting	1	$17.60
3	Ball Bearings	Ball Bearings	2	$16.08 Each
4	Pressure Regulator	Pressure Regulator	1	$283.26
5	Adjustable Relief Valves	Relief Valves	1 (20psi)	$8.24

6	Liquid N_2 Container (CT-50)	Liquid Nitrogen Container	1	$469

Table 2: Appendix A- Off the shelf components (cont...)

#	Component	Url	Quantity	Price
7	Self-Regulating Heat Cable	Heat Cable	1	$190.26
8	Gaseous N_2 Container	Gaseous Nitrogen Tank	1	$131.35
9	Nitrogen Gas	Nitrogen Gas Refill of Tank	1	$2.90
10	Hose with Tube Connections for Liquid Nitrogen	Hose w/ Tube	3 ft.	$191.40
11	Type 304 Stainless Steel Threaded Pipe L Fitting	Pipe Fitting	5	$4.90 Each
12	Type 304 Stainless Steel Threaded Pipe T Fitting	Threaded Pipe Fitting	1	$6.91

13	Pan Head Phillips Machine Screws	Machine Screws	Pkg. Qty 50	$6.32
14	Rubber handles	Rubber Handles	Pack of 6	$13.79

Table 2: Appendix A- Off the shelf components (Cont…)

#	Component	Url	Quantity	Price
15	Warning Sticker	Warning Sticker	1 / Device	$7
16	Thermocouples	Thermocouple	2	$29.64 each
17	Electrical Wire	Electrical Wire	10ft	$8.32
18	Type 316 Stainless Steel Yor-Lok Tube Fitting	Yor-Lok Tube Fitting	1	$20.55

23	LCD screen	Sparkfun Small LCD	1	$24.95
24	MicroController	Adafruit Feather	1	$ 29.95
27	Micro USB cable to power MicroController	Remax Smart Charger	1	$4.85
28	Buttons	Buttons	20 Pack	$2.50
Off the shelf components			Total price	$ 1,552.07
Estimated Soup Container Materials and Production Cost				
Stainless Steel Metal		Stainless Steel Metal	50 lb	$7
Welding per hour		Welder	~5 hours	$12.71/ hr
Machining per hour		Machinist	~10 hours	$12.40/ hr
Soup Container			Total price	$194.55

Total Price of System is ~ $1,746.62