Splintered Twister
Project 3 Report

Project Group 3

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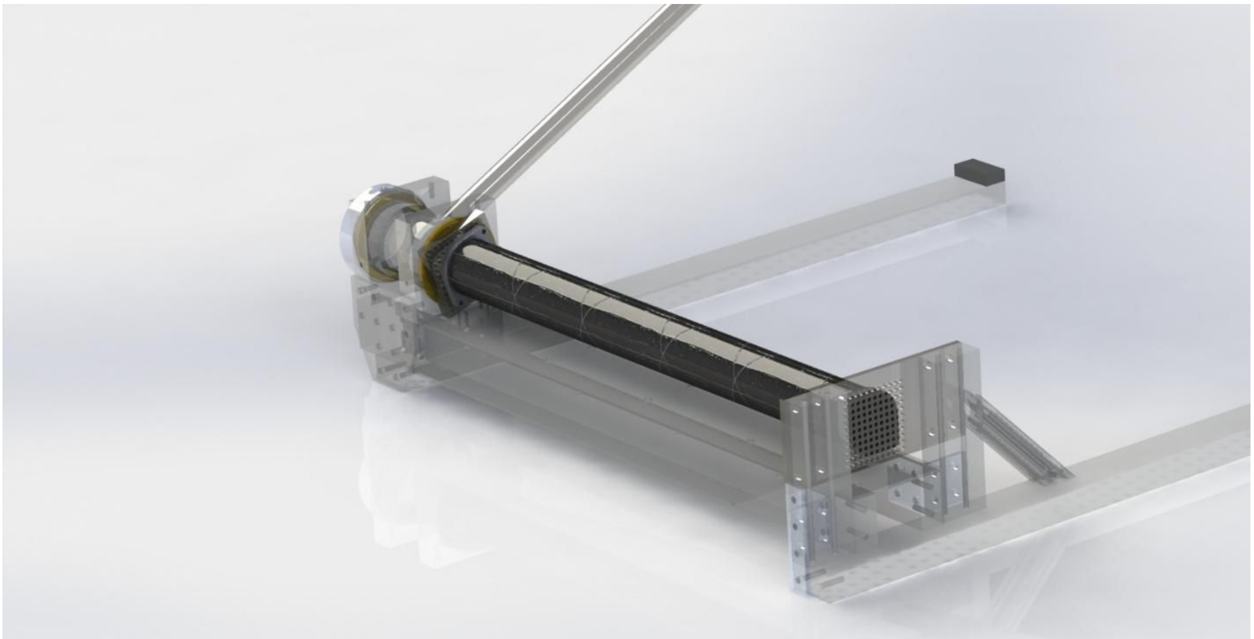
Summary



The design consists of three main components: the composite tube and the two end pieces, attached rigidly using an epoxy adhesive. The tube is composed of filament wound carbon fibers laid in patterns $45°$ from the central axis to optimize torsion transmission. The tube has an outer diameter of $2.186\ in$ and an inner diameter of $2\ in$ with a total length of $24\ in$. The ends of the tube are then attached using an epoxy adhesive to cylindrical endpieces to a depth of $0.35\ in$ into the tube length. These endpieces are then fixed onto the pegboard of the testing stand by flanges with clearance holes for 10-24 screws.

A typical carbon fiber rod optimized for bending or tensile loads has plies oriented along the length of the tube as well as its radius. Our tube contains plies oriented $45°$ from this configuration; rod's stiffness and strength reduces to bending and tensile loads, but increases for torsional loads. As our component undergoes torsional loads only, this orientation is optimal for our design.

We choose to use an epoxy adhesive to fix the tube to the endpieces as the plies are already constructed using epoxy. To ensure that the part does not fail at these joints, we used the 3M DP420 epoxy adhesive with a very high strength. Furthermore, to ensure that the endpieces fully constrain the tubs and transmit the torque without yielding, we use two 10-24 screws to fasten the endpiece onto the testing stand.

The total mass of the assembly is roughly 359.3 grams. The carbon fiber tube is estimated to be 319.3 grams. The mass of each endpiece is 20.0 grams. We believe the mass of the epoxy adhesive between the tube and endpiece is negligible.

The lowest factor of safety in our design is 1.12. The angular deflection of the rod is estimated to be $8.968°$ and the torque applied varies linearly with the angular deflection, causing the factor of safety to be roughly 1.12.

The primary mode of failure is excessive angular deflection along the length of the tube.

Fundamentals

The central load in this design is the $200\ Nm$ torque applied by the testing rig, which induces shear loads – more specifically, torsional loads – on the carbon fiber rod. We must also consider the loads on the endpieces and fasteners used to constrain the rod, and the central load for these components is also shear. The failure criteria for the tube is when the part yields due to shear stress or when it suffers angular deflection larger than $10°$.

$$\tau_{rod} = \frac{TR}{J} \cong \frac{4T}{2\pi t D_o^2} \leq \tau_{max,rod}$$

$$\theta = \frac{LT}{JG} \cong \frac{4LT}{\pi t D_o^3 G} \leq \theta_{max}$$

Where T represents the torque applied, D_o represents the outer diameter, t represents the thickness, L represents the length of the rod, G represents the shear modulus, $\tau_{max,rod}$ represents the maximum shear strength of carbon fiber, and θ_{max} represents the maximum allowable angular deflection.

For the endpieces and fasteners, we only consider yielding due shear stress.

$$\tau_{epoxy} = \frac{F}{A} = \frac{2T}{\pi D_o^2 w} \leq \tau_{max,\ epoxy}$$

$$\tau_{fastener} = \frac{F}{nA} = \frac{4T}{n x_{fastener} \pi D_{fastener}} \leq \tau_{max,fastener}$$

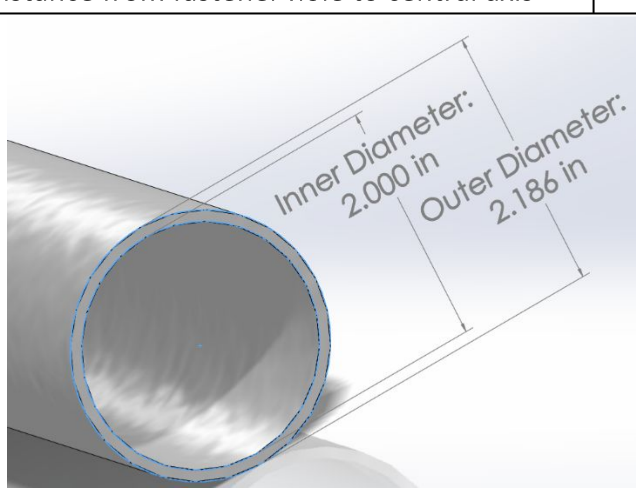
Where w represents the contact length with the carbon fiber tube, $\tau_{max,epoxy}$ represents the maximum shear strength of the epoxy used to fix the rod to the endpieces, n represents the number of fasteners, $x_{fastener}$ represents the distance from the fasteners to the central axis, and $D_{fastener}$ represents the diameter of the fastener.

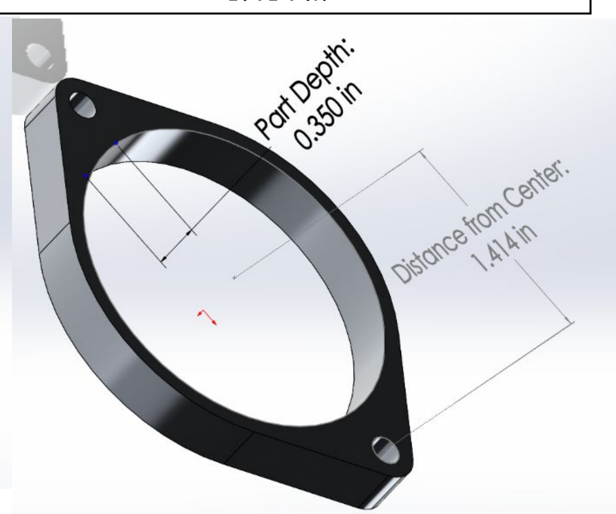
These analyses led us to determine appropriate inner and outer diameters for the carbon fiber tube:

Inner Diameter	$2.186\ in$
Outer Diameter	$2.000\ in$

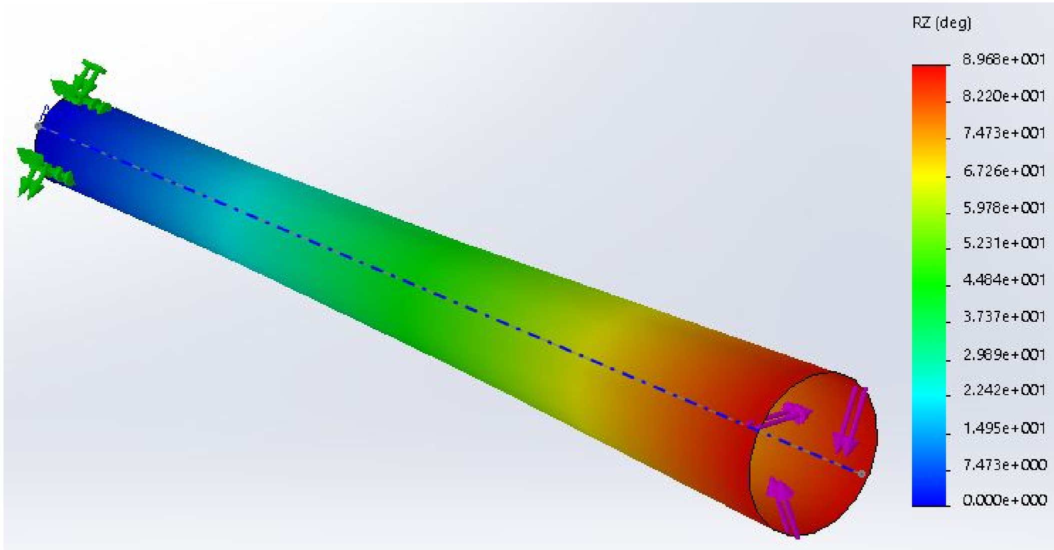
And key endpiece dimensions:

Part Depth (how much it contacts the tube)	$0.350\ in$
Number of fasteners used (Number of clearance holes for fasteners)	2
Distance from fastener hole to central axis	$1.414\ in$

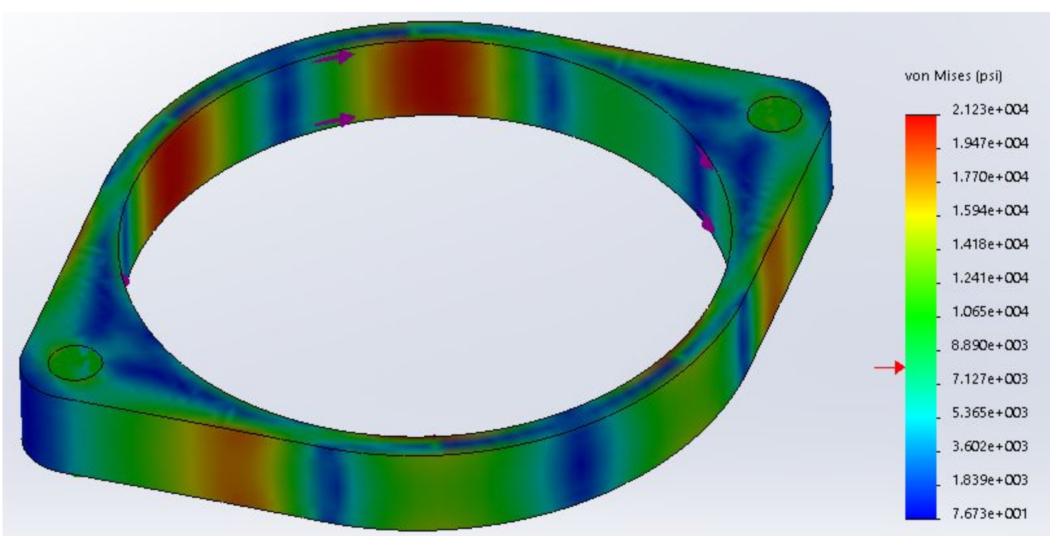




CAD Analyses



The carbon fiber rod was modeled using a surface extrusion and a composite definition in a Static analysis. We found a typical ply thickness value of ~ 0.0155 inches[(4)] and due to the thickness of the rod being 0.093 inches, we estimate roughly 6 plies of the composite material. To run the analysis, we fixed one end of the rod up to the depth of the endpiece and applied the torque on the other end up to the same depth. Our analyses indicate that the rod does not fail nor deflect more than 10° under testing conditions with a factor of safety of 1.12. Hand analyses indicated that the angular deflection constraint was more constraining than the shear stress constraint, and the CAD analysis supports this claim.



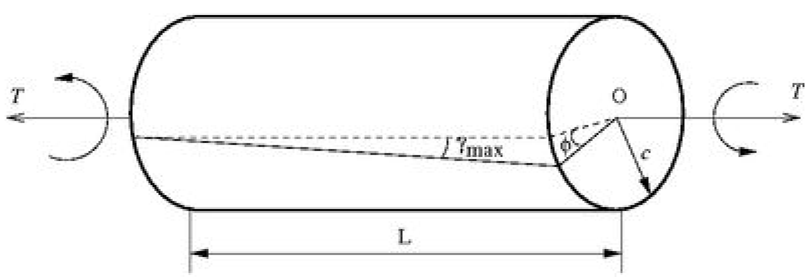
We also performed CAD analysis on the endpieces for the tube. To perform these analysis, we input pin joints on the two fastener holes and a torque on the inside face of the endpiece (which contacts the tube). Our analysis indicates that the piece will not break under the torsional load when constrained in this manner, with a factor of safety of 1.88.

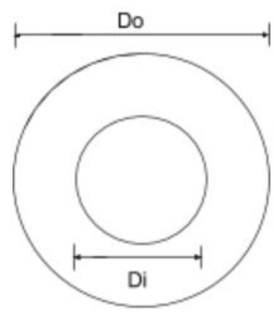
Appendix

Detailed Part Analysis

Our part can be modeled as three components: the (carbon fiber) rod under torsion and the two (aluminum) endpieces that fix the rod onto the faces of the testing stand. We perform separate analyses on these parts in order to simplify the calculations.

Detailed Tube Analysis





Known Parameters / Constraints	Free Parameters
$Torque\ Applied\ (T) = 200\ Nm$ $Maximum\ Deflection\ (\theta_{max}) = 10°$ $Length\ (L) = 24\ in$ $Yield\ Strength\ (\sigma_y)$ $Modulus\ of\ Rigidity\ (G)$ $Density\ (\rho)$	$Tube\ Outer\ Diameter\ (D_o)$ $Tube\ Inner\ Diameter\ (D_i)$

We want to minimize the mass of this torsion rod; therefore, our mass is the objective function of this analysis.

Objective Function:

$$m = \rho V = \frac{\rho L \pi}{4}(D_o^2 - D_i^2)$$

$$m = \frac{\rho L \pi}{4} * (2D_o t)\ (for\ a\ thin - walled\ tube)$$

Since the rod must neither break nor deflect by more than θ_{max} under the torsional load, we perform separate analyses for yield due to shear and yield due to angular deflection.

Maximum Shear Stress:

$$\tau = \frac{TR}{J}$$

$$R = \frac{D_o}{2} \ (Since\ shear\ stress\ is\ maximum\ on\ the\ outer\ surface\ of\ the\ rod)$$

$$J = \frac{\pi(D_o^4 - D_i^4)}{32}$$

$$\tau \leq \tau_{max} = \frac{\sigma_y}{2} \ (for\ torsion)$$

$$\rightarrow \frac{\sigma_y}{2} \geq \frac{32TD_o}{2\pi(D_o^4 - D_i^4)}$$

$$\rightarrow \frac{D_o}{D_o^4 - D_i^4} \leq \frac{\pi\sigma_y}{32T}$$

This equation is more useful if we write it in terms of the outer diameter and wall thickness.

$$J = 2\pi t (R_{avg})^3 \ (for\ a\ thin-walled\ tube)$$

$$\rightarrow J = \frac{1}{4}\pi t D_o^3$$

$$\rightarrow \frac{\sigma_y}{2} \geq \frac{4TD_o}{2\pi t D_o^3}$$

$$\rightarrow D_o - \sqrt{\frac{4T}{\pi t \sigma_y}} \geq 0$$

This provides the shear strength constraint of the rod. Note that this system alone is unsolvable because there are two unknown variables $(D_o,\ t)$.

Maximum Angular Deflection:

$$\theta = \frac{LT}{JG} \leq \theta_{max}$$

$$J = \frac{\pi(D_o^4 - D_i^4)}{32}$$

$$\rightarrow \theta = \frac{32LT}{\pi(D_o^4 - D_i^4)G} \leq \theta_{max}$$

$$J = \frac{1}{4}\pi t D_o^3 \ (for\ a\ thin-walled\ tube)$$

$$\rightarrow \theta_{max} - \frac{4LT}{\pi t D_o^3 G} \geq 0$$

$$\rightarrow D_o - \sqrt[3]{\frac{4LT}{\pi t \theta G}} \geq 0$$

With the objective function of the mass:

$$m = \frac{\rho L \pi}{4} * (2D_o t) \ (for\ a\ thin-walled\ tube)$$

And the two constraints of the system:

$$D_o - \sqrt{\frac{4T}{\pi t \sigma_y}} \geq 0$$

$$D_o - \sqrt[3]{\frac{4LT}{\pi t \theta G}} \geq 0$$

We ran a nonlinear solver using Matlab's fmincon minimizer function to solve for D_o and t values that minimize the objective function given the two constraints. Other constraints inputted into this minimizer included D_o and t both being positive and that t is strictly less than D_o.

```
T = 200;
theta_max = 10;
L = .6096;
density = 1550;
% z = do, t
mass = @(z) density * L * (pi / 4) * (2 * z(1) * z(2));

fmincon(mass, [0 0], [], [], [], [], [0 0], [], @confuneq);

function [c, ceq] = confuneq(z)
T = 200;
sig_y = 137895 * 10^3;
G = 31026.4 * 10^6;
L = .6096;
theta_max = .1745;
c(1) = - sqrt(4 * T/ (pi * z(2) * sig_y)) + z(1); % <= 0
c(2) = - ((4 * L * T)/(pi * z(2) * theta_max * G))^(1/3) + z(1); % <= 0
c(3) = z(2) - z(1); % <= 0
ceq = []; % = 0
```

The results of the minimizer seemed to indicate that a tube with a large enough outer diameter D_o and a thin wall thickness t such that the constraints were satisfied would have the minimal mass. Using this information, we decided that a large diameter carbon tube with a very thin wall thickness would be the optimal solution. Though Matlab provided us with exact values for the dimensions that would minimize mass while still maintaining the constraints, our parameters were limited by the products that were available online; that is, manufacturers of carbon tubes have set diameters and thicknesses that they sell. While custom tube manufacturing is possible, the long wait time for ordering a custom tube (typically 10 weeks) made it not an option.

Our team decided on a 24" carbon fiber tube with a 2.186" outer diameter and 2" inner diameter from Rockwest Composites(3). These dimensions meet the constraints of the system outlined previously with a generous factor of safety while optimized for minimal mass.

One important aspect of our calculations to note is that material properties for the carbon fiber tube were not easily available on the Rockwest Composites website. To run our analyses, we had to estimate the material properties of the composite using information about the carbon fibers themselves and the epoxy used.

Standard Modulus Carbon	Tensile Modulus (stiffness): 20,000,000 psi(1) Ultimate Tensile strength: 350,000 psi(1)
Lindride 6K Hardener (Hardener for Epoxy Resin)	Tensile Strength: 12,000 psi(2) Tensile Modulus: 446,000 psi(2)

Using this information, we estimated that the yield strength of the carbon fiber tube is roughly 350,000 psi and that the shear modulus is roughly 446,000 psi. We believe these estimates to be relatively accurate due to the layup structure of our tube: our particular carbon fiber tube is specialized for torsion, and the carbon fibers are laid 45 degrees from its central axis.

Detailed Endpiece Analysis

The general design of our endpiece is a cylindrical component that contacts the tube to a depth w. In order to attach the endpieces to the testing stand faces, we rely on a grid of threaded mounting holes for 10-24 screws; therefore, we must create attachment fixtures on the endpieces with clearance holes for these 10-24 screws. In order to fully constrain the endpiece as well as prevent it from yielding under the shear load, we create n flanges on the endpiece each with a clearance hole for the 10-24 screw. The final free parameter is the distance between the central axis of the endpiece and the center of the fastener holes, $x_{fastener}$.

Known Parameters / Constraints	Free Parameters
$Torque\ Applied\ (T) = 200\ Nm$ $Shear\ Strength\ of\ Epoxy\ (\tau_{max,epoxy})$ $Shear\ Strength\ of\ Fastener\ (\tau_{max,fastener})$ $Outer\ Diameter\ (D_o)$ $Diameter\ of\ the\ Fastener\ (D_{fastener})$	$Part\ Depth\ (w)$ $Number\ of\ attachment\ points\ (n)$ $Distance\ from\ center\ (x_{fastener})$

To choose w, we need to consider the epoxy that rigidly connects the endpiece to the tube. We need to make sure that the shear stress induced by the torque onto the epoxy will hold for the surface area of epoxy applied. This surface area of epoxy is dependent on the outer diameter of the rod (derived in the previous analysis) as well as w. We chose to use 3M DP420 Epoxy Adhesive, which has a shear strength of roughly $4500\ psi$ (5).

$$\tau_{epoxy} = \frac{F}{A} \leq \tau_{max,\ epoxy}$$

$$\frac{\frac{T}{D_o}}{\frac{\pi}{2} D_o w} = \frac{2T}{\pi D_o^2 w} \leq \tau_{max,epoxy}$$

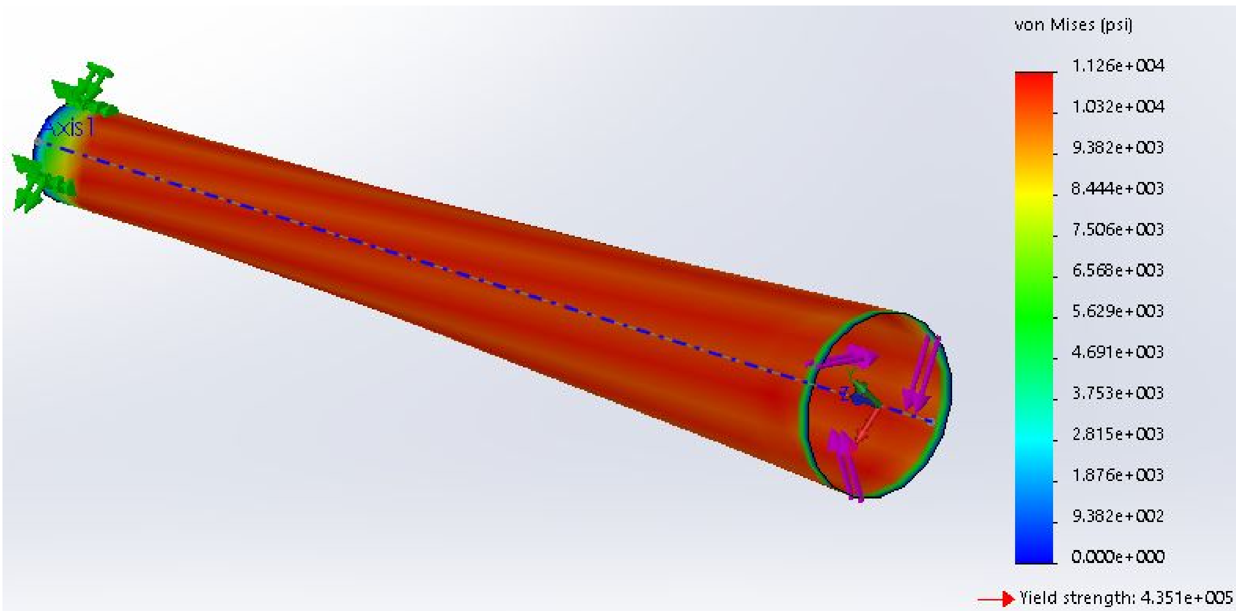
As w is the only unknown in this equation, we simply solved for the minimum value of w that would meet this constraint. This value was found to be relatively small at $0.052\ in$. Since we are not sure that the epoxy will completely fill all the small gaps between the tube and endpiece, we chose a very high factor of safety of roughly 7 and set w to be $0.35\ in$.

We further performed analysis on the fasteners to determine the number of fasteners to use as well as the distance between the fastener and the center of the endpiece. The 10-24 screws we will use in testing are made of 18-8 Stainless Steel, with a shear strength of roughly 37,500 psi at the lowest estimates. As our tube and endpieces are relatively large, we were limited in the pegs we could use for the fasteners – we chose the ones in the very corners of the pegboard, which is equal to $x_{fastener} = \sqrt{2} \cong 1.414\ in$.

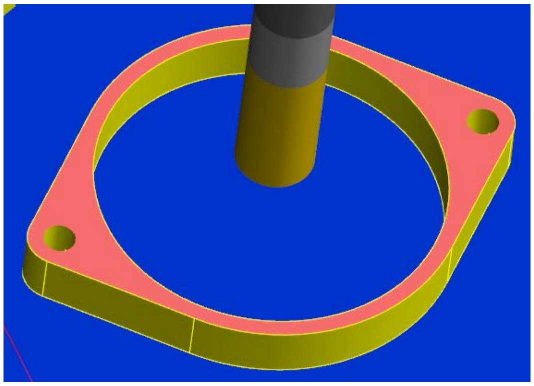
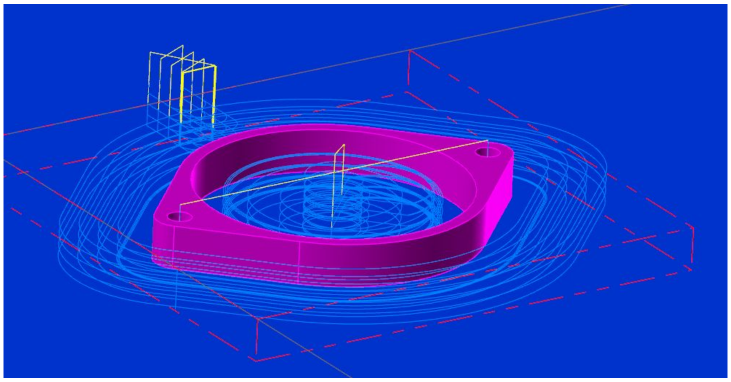
$$\tau_{fastener} = \frac{F}{nA} = \frac{4T}{n x_{fastener} \pi D_{fastener}} \leq \tau_{max,fastener}$$

We could then solve for the minimum number of fastener holes needed to constrain the system, which is two.

Additional CAD Diagrams and Results



We find that the tube does not yield and break under the load.

Setup for machining endpieces

Works Cited

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